UNIVERSITY OF APPLIED SCIENCES UTRECHT

Using reality in mathematics education: Exploring new opportunities

RME6, Georgetown, Cayman Islands
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Saturday, 22 September 2018

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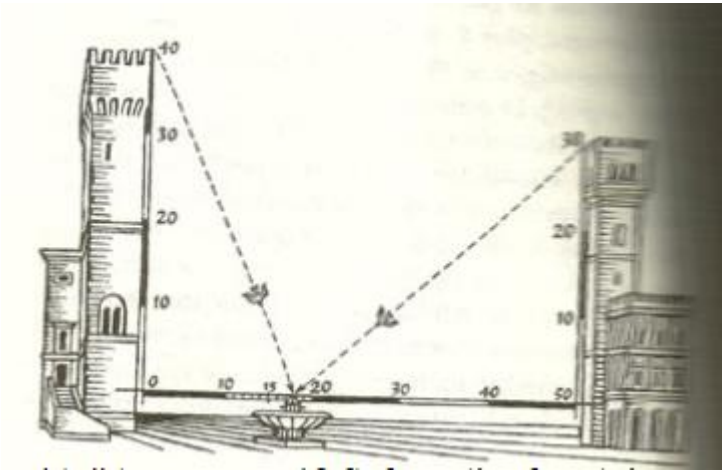
Three experiences

- Images
- School example
- History



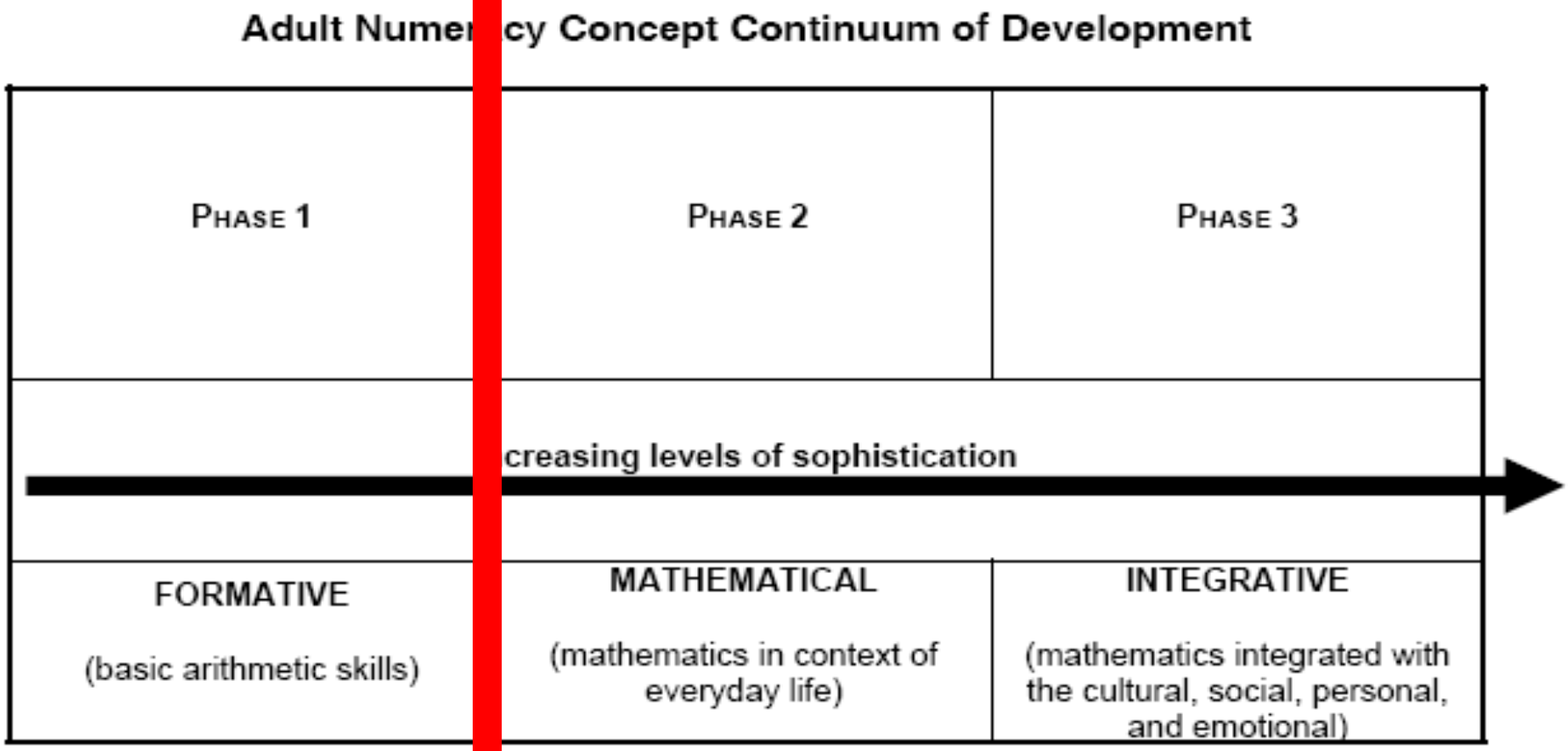
On two towers, 30 and 40 feet high, birds are sitting. On a certain time both fly with the same speed to a foutain that is on the straight line between the two towers (or outside the towers).
Find the position of the fountain if the distance between the towers is 50 feet.

Source: Leonardo of Pisa (Fibonaci), 1202, Liber Abaci

Overview of presentation

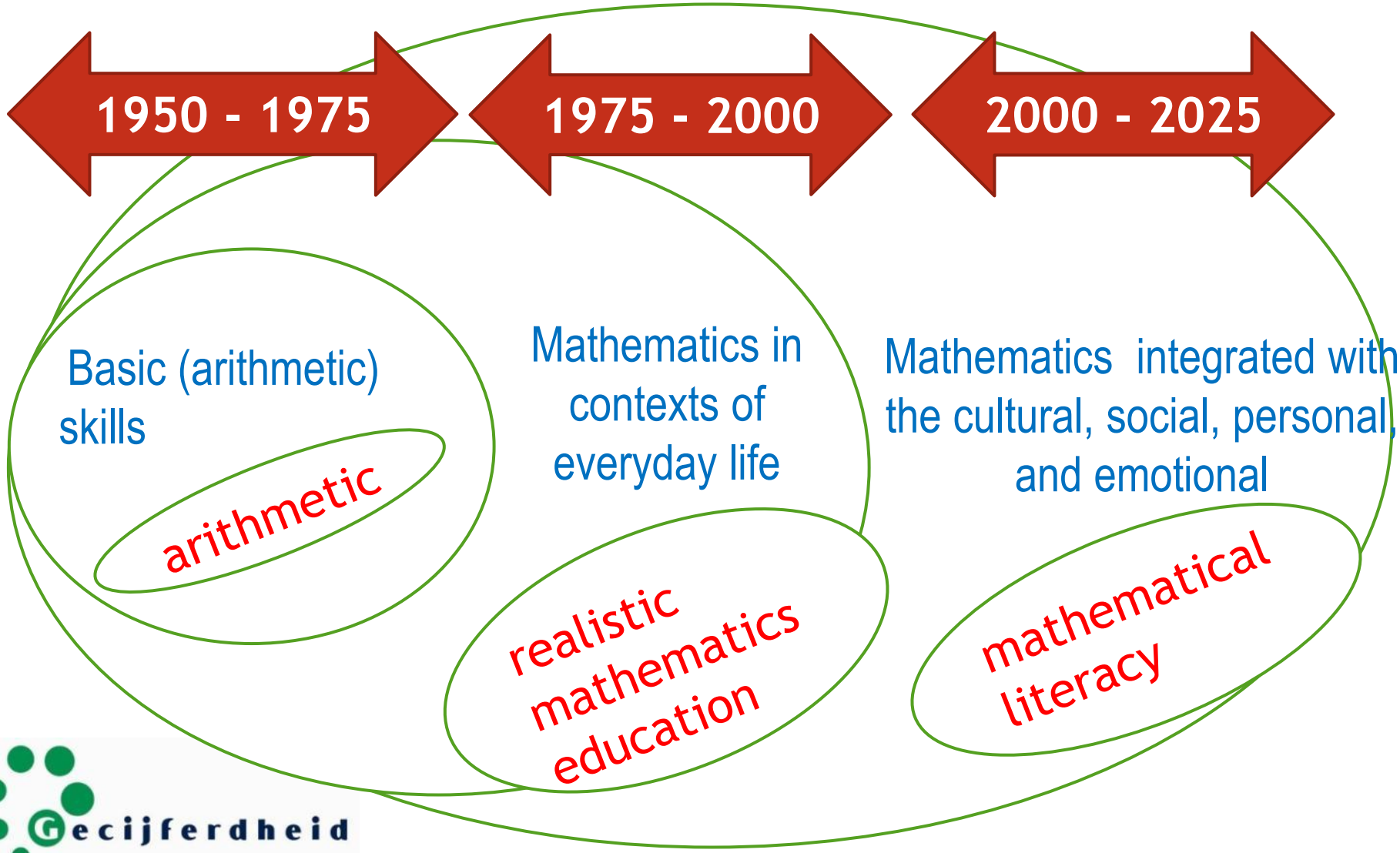
- Educational trends and the 20th century dominance of word problems
- The use of reality in RME
- Some research results
- Exploring new opportunities to use reality in mathematics education
  - Item framework

Paradigmatic barrier
Epistemological shift
Back-to-the-basics



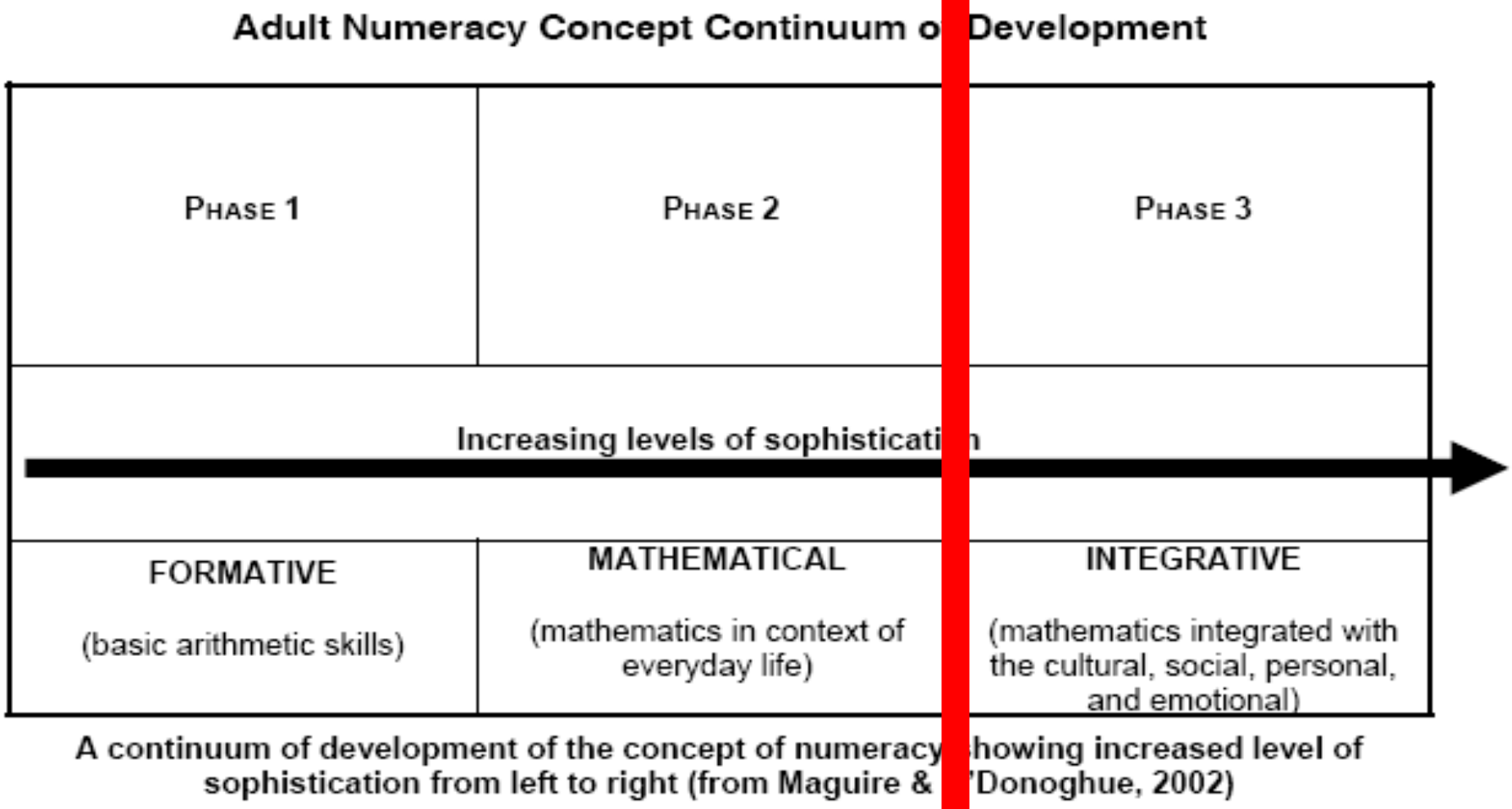
A continuum of development of the concept of numeracy showing increased level of sophistication from left to right (from Maguire & O'Donoghue, 2002)

Numeracy & Functional Mathematics
Conceptual development



Gecijferdheid

Paradigmatic barrier ???
Epistemological shift ??
Lack of imagination ??



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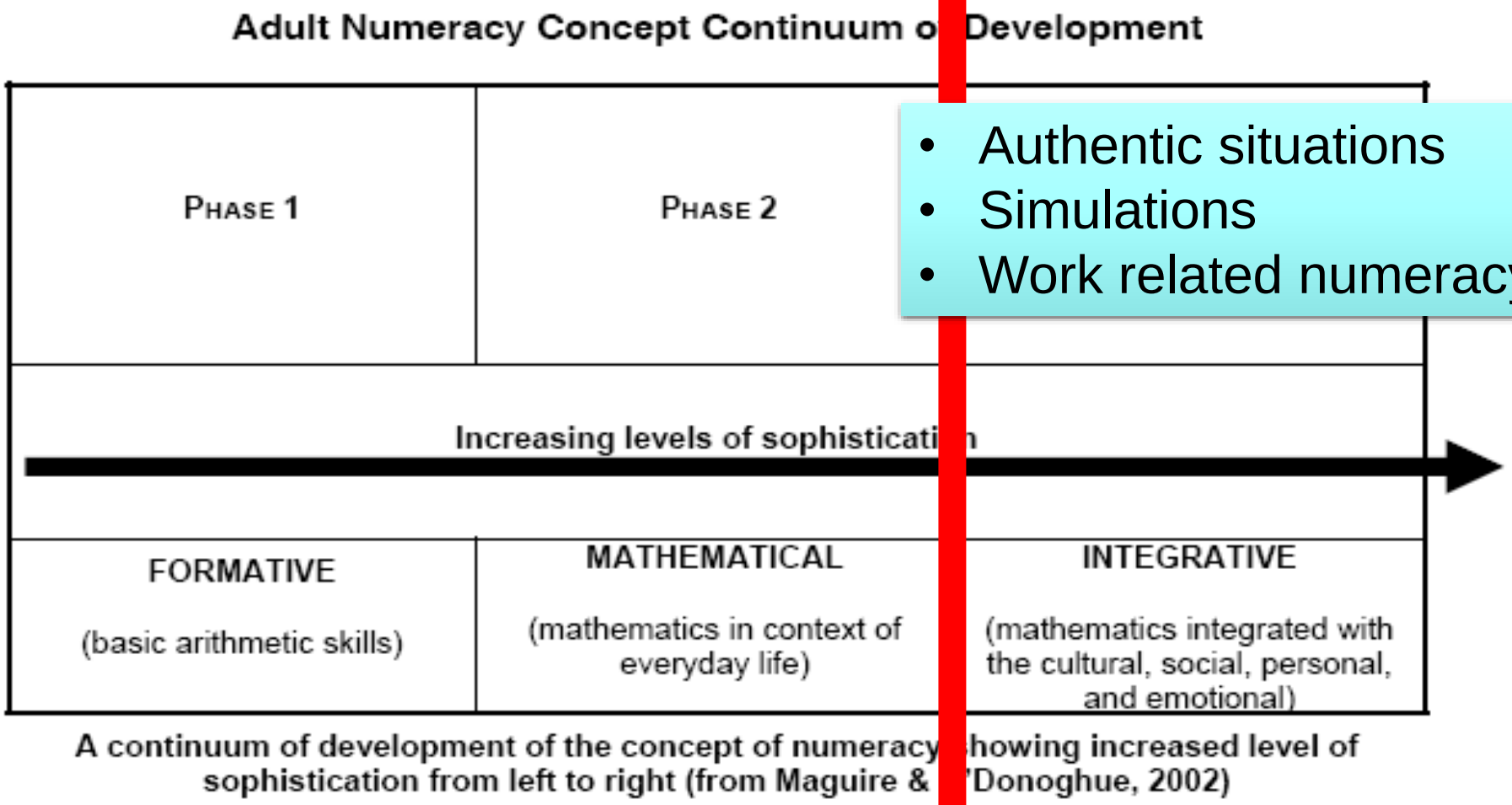
Assessment

1 2 3

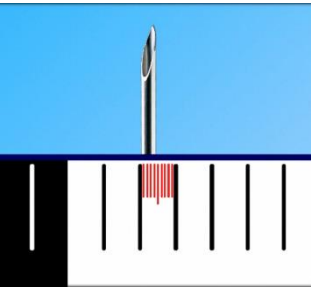


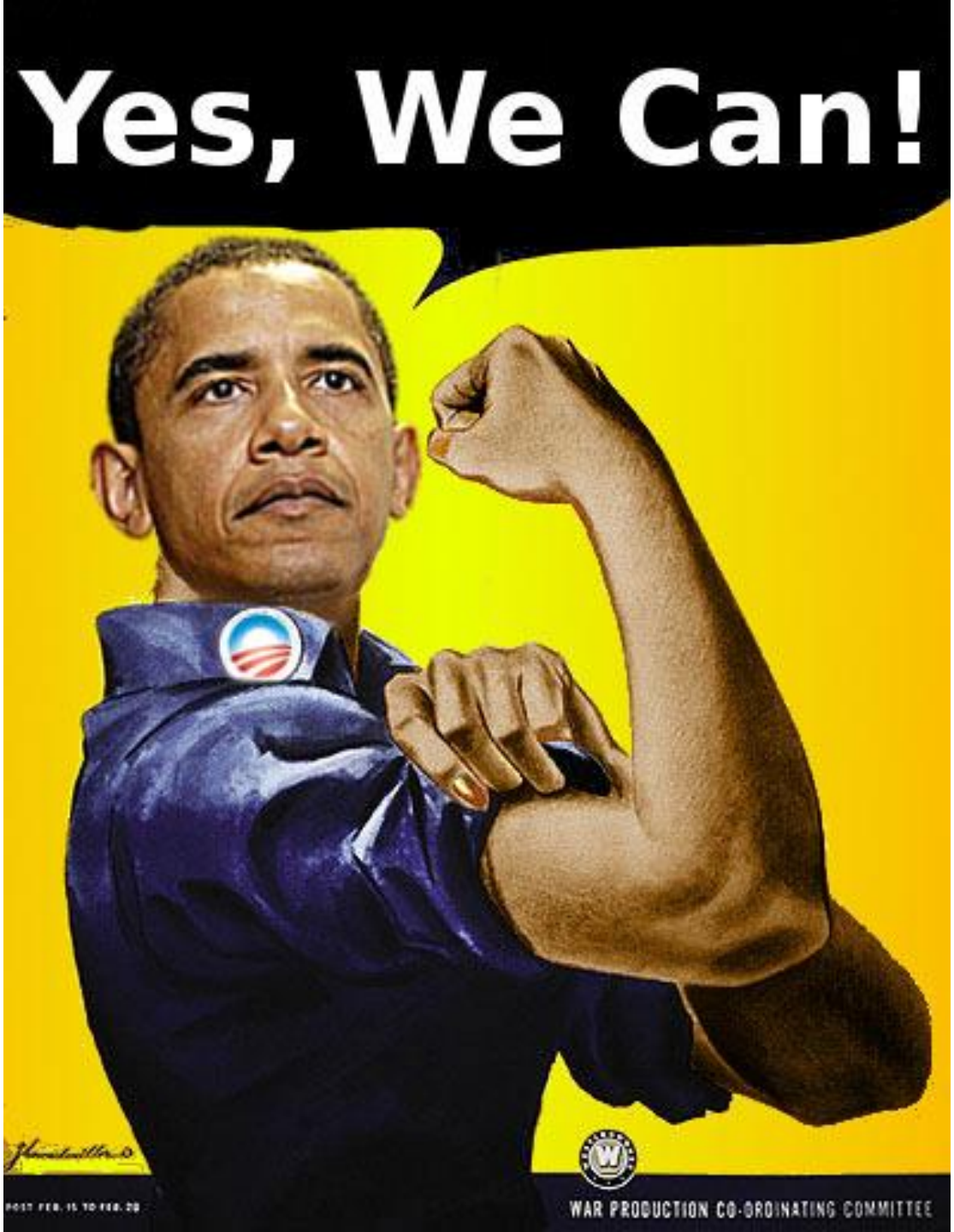
- Authentic situations
- Simulations
- Work related numeracy

Some serious troubles with word problems as a tool to use reality

- Language problems
  - Contextualising, recontextualising
- “Hidden curriclum” problems
- Suspension of sense making
  - Socio-mathematical norms

Can we improve “regular“ lesson materials and “regular” tests to fit more sophisticated numeracy concepts?




Yes, We Can!
WAR PRODUCTION CO-ORDINATING COMMITTEE

Example

- In the real world multiplication is about multiplication structures (and only sometimes repeated addition)
- Test

- Improving PISA items

Apples are sold in 2.5 kilogram bags.
You weigh one apple and find 157 grams.

About how many apples are there in one bag?

[ ] apples

You buy groceries for a total of € 21.30
You pay with a 50 euro bill and two coins of 1 euro.

How much is the change?
€ [ ]

The bath room has two windows.
They both are 0,90 m in width and 1,35 m in height.
You want to double glaze these windows.
Double glazing costs € 148,- per m^2

What is the cost of double glazing these windows?
€ [ ]

About how many apples are there in one bag?

[ ] apples

Je moet betalen

You have to pay:

SUPERMARKT

Daliastraat 4
5707 SJ Helmond
0492-527384

15	blik cola 330ml	0.90	13.50
13	chips flav. pnt. light	0.60	7.80
aantal art. 28		subtotaal	21,30
TOTAAL			21.30

Je betaalt met

You pay with:

How much is the change?

€ [ ]

dubbelglas

€148,- per m²

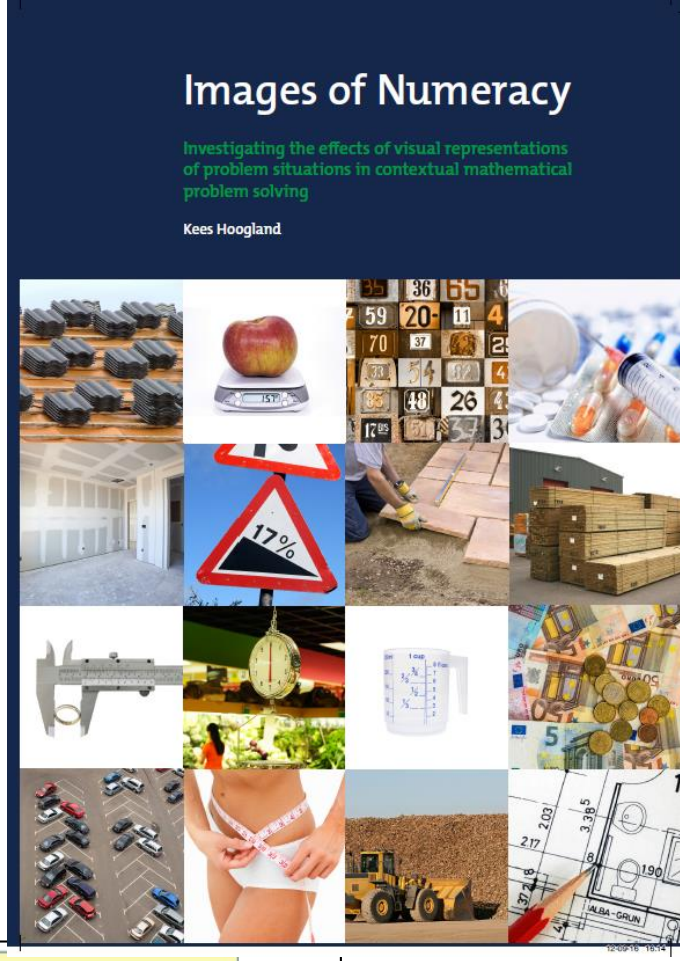
Double glazing
€ 148.- per m²

What is the cost of double glazing these windows?

€ [ ]

Images of Numeracy

Investigating the effects of visual representations of problem situations in contextual mathematical problem solving



From a descriptive representation of reality to a depictive representation of reality.

Je koopt boodschappen voor € 21,30.
Je betaalt met een biljet van 50 euro en twee munten van een euro.

Hoeveel krijg je terug?
€

Data

- Main run
  - 179 schools
  - 31,842 students
  - Primary (11-12 yr)
    - 1,150
  - Secondary (12-16/18 yr)
    - 29,500
  - Sec. vocational (16-20 ytr)
    - 1,000

- Collected data
  - Scores on items
  - Answers to items
  - Grade level
  - School track level
  - Age
  - SES
  - Gender
  - Ethnicity
  - Time spent on items in ms
  - Last math grade

Large scale testing as a numeracy test:

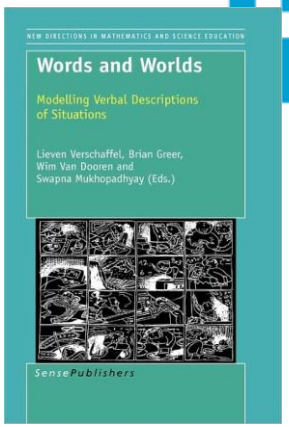
random 12 A + 12 B, random order

controlled randomized trial

Theoretical notions

- Contextual (word) problems & their difficulties

  (Boaler, 1993), (Gravemeijer, 1994),
  (Verschaffel et al. 2000, 2009), Greer (1997)
  and many more



- Cognitive psychology
  - descriptive versus depictive representations

    (Schnotz et al., 2002, 2010)

- Cognitive load theory

  (Sweller, 1996, 1999; Mayer, 2005)

- "Answer getting mindset" versus "Problem solving mindset"

  (Boaler, 2016), (Daro, 2013)

Statistical analysis

Probit – analysis is a sophisticated multivariate analysis

How do the variables v (version) and x (others) contribute to the ability y.
The chance P(z = 1) that the question is solved correctly equals
the chance P(y ≥ δ) that ability y surpasses a treshold δ.

$$y = \alpha_0 + \alpha_1 v + \alpha_2 x + \varepsilon$$

$$z = 1 \text{ if } y \geq \delta$$
$$z = 0 \text{ otherwise}$$

$$P(z = 1) = P(y \geq \delta) = P(\varepsilon \geq -\alpha_0^* - \alpha_1 v - \alpha_2 x)$$
$$P(z = 0) = 1 - P(y \geq \delta)$$
$$with\ \alpha_0^* = -\delta + \alpha_0$$

v = {0 = descriptive repr., 1 = depictive repr. }
x_1 = {0 = male,1 = female}
x_2 = {0 = not, 1 = migrant family}
x_3 = {relative age within level grade}
x_4 = {last school math grade}
x_5 … x_9 = {school track level}

E = not observed variables (m=0, s=1)

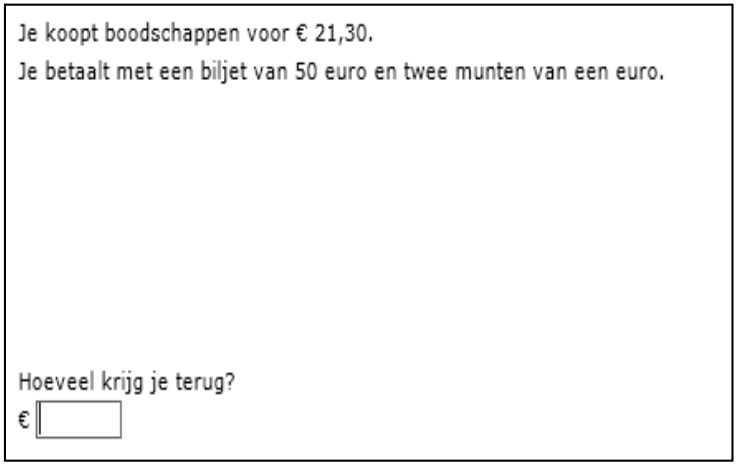
From the data the parameters α_0^*, α_1, α_2 are estimated by maximum likelihood.

Results

- B > A statistically significant, with a (very) small effect size.
- B > A on a significant number of problems (11/21)
- Bigger effect in domain of measurement & geometry

- Further research
  - Interdependency on other variables
  - Actual student behavior
  - Eye-tracking
  - Teaching focus

>



Results

		Model 1	Model 2	Model 3
Independent variables		marginal effect (std.err.)	marginal effect (std.err.)	marginal effect (std. err.)
v	variant (verbal/image-rich)	0.0207(0.0013)(*)	0.0207(0.0013)(*)	0.0197(0.0025)(*)
x_1	gender	0.0524(0.0013)(*)	0.0509(0.0013)(*)	[illegible]
x_2	etnicity	-0.0288(0.0015)(*)	-0.0267(0.0016)(*)	-0.0266(0.0022)(*)
x_3	age (relative)		-0.0124(0.0010)(*)	-0.0124(0.0010)(*)
x_4	math grade		0.0208(0.0005)(*)	0.0208(0.0005)(*)
x_5	primary education (rel.)	-0.1900(0.0034)(*)	-0.2183(0.0037)(*)	-0.2185(0.0052)(*)
x_6	pre-vocational (rel.)	-0.1810(0.0015)(*)	-0.1856(0.0016)(*)	-0.1863(0.0022)(*)
	general secondary	reference		
x_7	secondary vocational (rel.)	0.0859(0.0035)(*)	0.1020(0.0038)(*)	0.1006(0.0053)(*)
x_8	pre-university (rel.)	0.1136(0.0017)(*)	0.1029(0.0018)(*)	0.1021(0.0025)(*)
x_9	school level	0.0845(0.0007)(*)	0.0899(0.0008)(*)	0.0[illegible](*)
v * x_2	variant *etnicity			-0.0001(0.0032)
v * x_5	variant * primary education			0.0006(0.0093)
v * x_6	variant * pre-vocational			0.0015(0.0032)
	variant * general secondary			reference
v * x_7	variant * secondary vocational			0.0029(0.0075)
v * x_8	variant * pre-university			0.0017(0.0035)
	N	646,275	605,430	605,430
	Pseudo R2	0.0647	0.0676	0.0676

Broader perspective

- Simulating "reality" in classroom context
  - Verbal / descriptive
  - Visual / photographs / depictive
  - Visual / video clips / animated
  - Visual / augmented reality /

ZDM
https://doi.org/10.1007/s11858-018-0944-2

ORIGINAL ARTICLE

Computer-based assessment of mathematics into the twenty-first century: pressures and tensions

Kees Hoogland[1] · Dave Tout[2]

Accepted: 9 May 2018

Abstract

In recent decades, technology has influenced various aspects of assessment in mathematics education: (1) supporting the assessment of higher-order thinking skills in mathematics, (2) representing authentic problems from the world around us to use and apply mathematical knowledge and skills, and (3) making the delivery of tests and the analysis of results through psychometric analysis more sophisticated. We argue that these developments are not pushing mathematics education in the same direction, however, which creates tensions. Mathematics education—so essential for educating young people to be creative and problem solving agents in the twenty-first century—is at risk of focusing too much on assessment of lower order goals, such as the reproduction of procedural, calculation based, knowledge and skills. While there is an availability of an increasing amount of sophisticated technology, the related advances in measurement, creation and delivery of automated assessments of mathematics are however being based on sequences of atomised test items. In this article several aspects of the use of technology in the assessment of mathematics education are exemplified and discussed, including in relation to the aforementioned tension. A way forward is suggested by the introduction of a framework for the categorisation of mathematical problem situations with an increasing sophistication of representing the problem situation using various aspects of technology. The framework could be used to reflect on and discuss mathematical assessment tasks, especially in relation to twenty-first century skills.

Keywords CBAM · Twenty-first century skills · Higher-order thinking · Assessment framework · Mathematical assessment tasks · Mathematics assessment · Technology

Increasing sophistication

Category:	A	B	C	D	E	F	G
Type of representation	Decontextualised mathematical problem (e.g. 25 – 3 = ...)	Simple contextual word-based problems (e.g. You have 25 sheep. Three are stolen. How many do you have left?)	More complex contextual problems with images and descriptive representations (but no interactivity)	Sophisticated static contextual problems with depictive representations and interactivity in **response** space, but no interactivity in **situation** space	Sophisticated dynamic contextual problems with short video clips or animations as representation and interactivity in **response** space, but no interactivity in **situation** space	More sophisticated **multimodal** contextual problems with interactivity in **both** the **situation** and **response** spaces	Content of all previous categories, with augmented or virtual reality, with simulation of real situations or in real situations. Full interactivity across an integrated **situation** and **response** space.
Possible delivery	Pen-and-paper CBA	Pen-and-paper CBA	Pen-and-paper CBA	CBA	CBA	CBA	CBA
Possible interactivity/ technological support available to student	Nil	Nil	Visual support (static only)	Visual support	Animations and Visual support	Animations, Visual support, Automatic calculation, Spatial & visual manipulation, Simulation of computer applications, Interactive graphing, etc.	Animations, Augmented/Virtual reality support, Automatic calculation, Spatial & visual manipulation, Simulation of computer applications, Interactive graphing, etc.
Possible automatically-scored response types	Multiple Choice Numerical field	Multiple Choice Numerical field	Multiple Choice Numerical field	Multiple Choice Numerical field, Click on, drag and drop, pull down menu, matching, ordering, etc.	Multiple Choice Numerical field Click on, drag and drop, pull down menu, matching, ordering, etc.	Click on, drag and drop, pull down menu, matching, ordering, manipulating fields to create a correct solution (e.g. spreadsheet,), digital working space with digital tools	Click on, drag and drop, pull down menu, matching, ordering, manipulating fields to create a correct solution, digital working space with digital tools, physical actions in simulations (e.g. choosing an object by grabbing it)
C21st higher-order thinking?	No	Minimal	Minimal	Some, with marking against rubrics	Some, with marking against rubrics	Yes, with marking against rubrics	Yes, with marking against rubrics

Fig. 2 The dimension of assessment possibilities. A framework depicting the increasing sophistication of the representation of mathematical problem situations

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C21st higher-order thinking?	No	Minimal	Minimal

E	F	G
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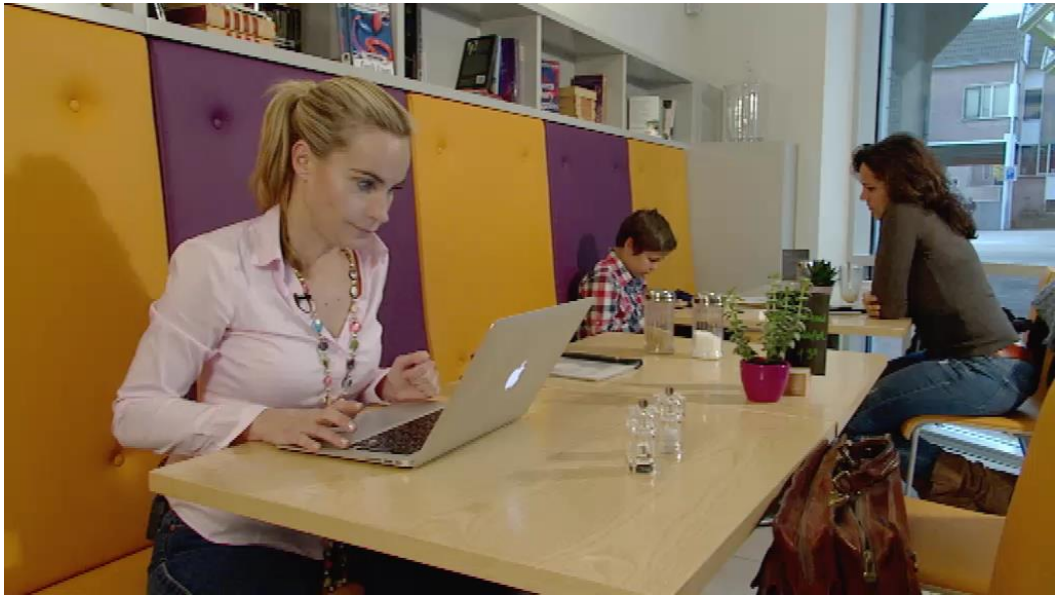
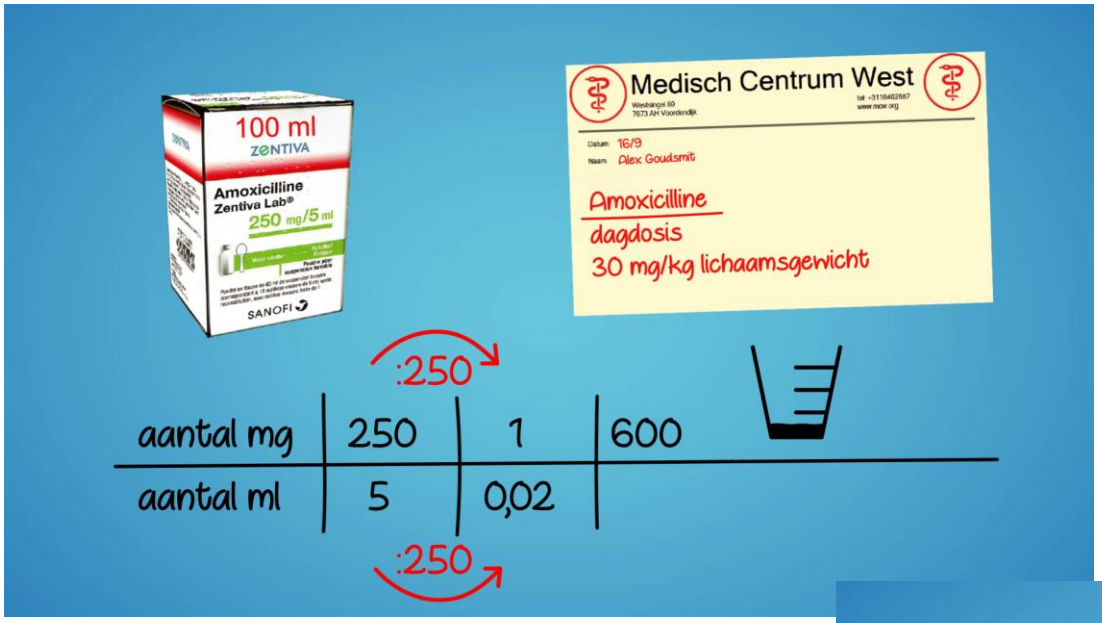
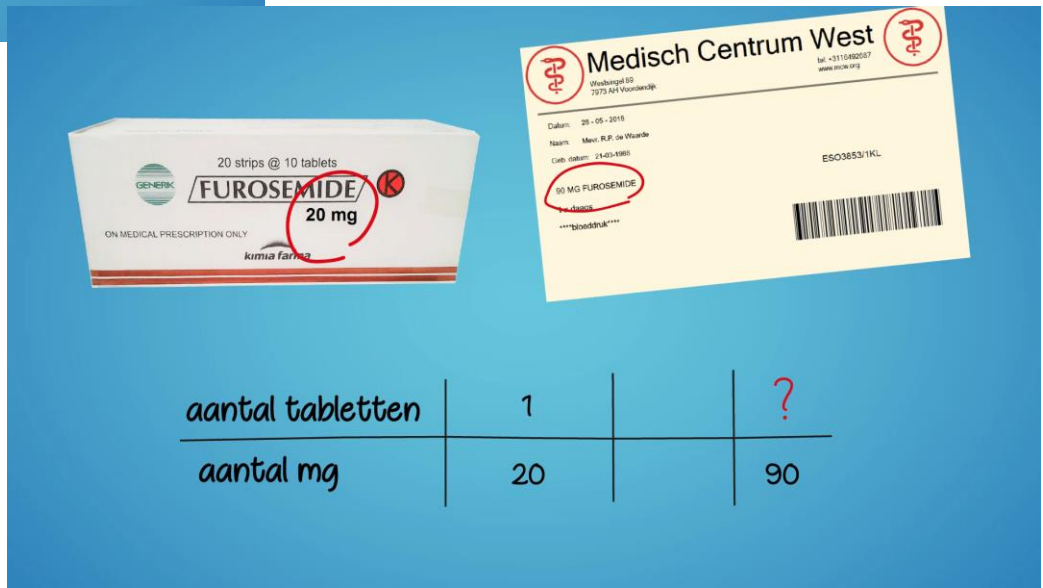
Videoclips



Video and animated explanation





Augmented reality

- Bringing reality into the classroom and activate the overlay.

Situation in or from reality

Overlay

Situation in or from reality

Technology: When the situation is detected, the overlay is activated

AURASMA

Education: When the relevant numeracy situation is detected, the educational overlay is activated

Augmented reality

- Bringing reality into the classroom and activate the overlay.

Situation in or from reality

Ov

Situation in or from reality

AURASMA

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Augmented reality

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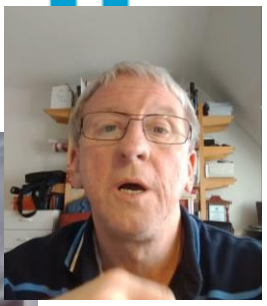

Situation i

Augmented reality

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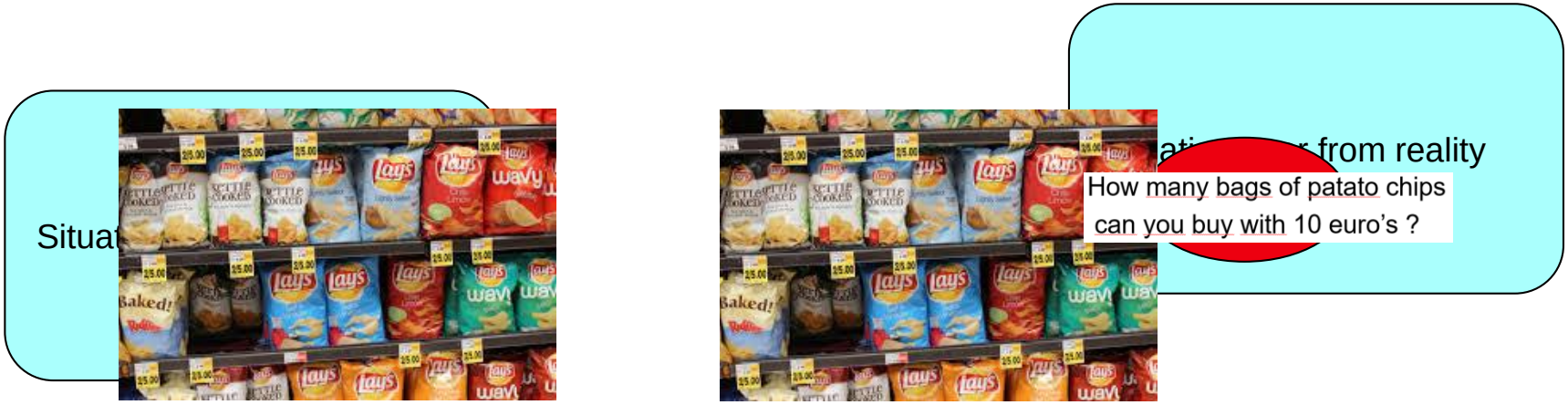

Technology: When the situation is detected, the overlay is activated

AURASMA

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Augmented reality

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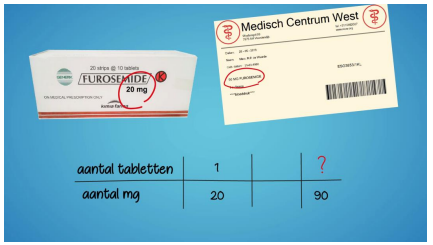
AURASMA



Education: When the relevant numeracy situation is detected, the educational overlay is activated.

Augmented reality

- Bringing reality into the classroom and activate the overlay.

Education: When the relevant numeracy situation is detected, the educational overlay is activated

AURASMA

Which App to use

To create (teacher or student):

Aurasma App (Mobile phone)

Aurasma Studio (PC)

To activate (student):

Aurasma / HP Reveal app

(Mobile Phone / Tablet)

Contact

Thank you for your attention !!

More information or suggestions?

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