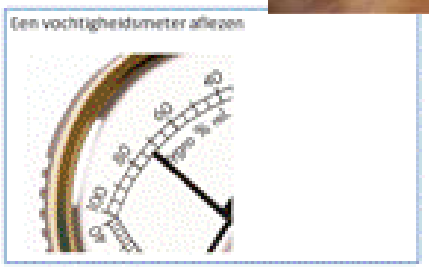
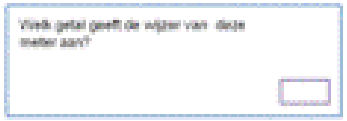
THE AMAZING USEFULNESS OF MATHEMATICS IN REALITY: A BASIS FOR MATHEMATICAL LITERACY





Intersection '17,
Makasar Indonesia
Saturday, May 6th, 2017

dr. Kees Hoogland, The Netherlands
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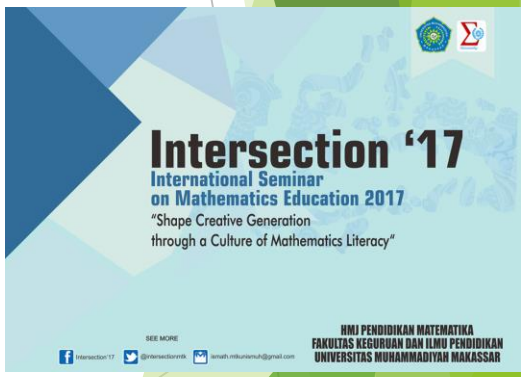

PRMI
M
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Two perspectives

- The relation between Mathematics & Reality
  - Modeling
  - Problem solving

- The role of Mathematics, Language & Images in Education
  - Research

Mathematics education

One of the major justifications of mathematics education is to equip students with knowledge and skills to solve practical quantitative problems (Niss, 1996).

9.68

a. $\dfrac{\frac{5}{6}+\frac{2}{5}}{\frac{2}{3}+\frac{1}{6}} =$

b. $\dfrac{\frac{3}{4}+\frac{4}{3}}{\frac{3}{4}-\frac{1}{3}} =$

c. $\dfrac{\frac{7}{8}+\frac{1}{3}}{\frac{4}{5}+\frac{1}{4}} =$

31 Jan knapt zijn slaapkamer op. Hij gaat de drie wanden lichtblauw verven. Hij heeft 5 liter verf gekocht, genoeg voor 30 m^2.

a De eerste muur heeft een oppervlakte van 4 m^2. Hoeveel verf heeft Jan voor deze muur nodig? Gebruik een verhoudingstabel.

b De volgende muur heeft een oppervlakte van 10 m^2. Heeft Jan dan genoeg aan 1 liter verf?

c De grootste muur is 14 m^2 groot. Bereken hoeveel verf hiervoor nodig is.

d Hoeveel liter verf is nodig voor 50 m^2?

ICME 2012 + 2016 - trends

- Professional development of teachers
  - Pedagogical content knowledge
  - Specialized content knowledge
    - specialized for teaching
- National improvement & (inter)national assessment
  - High stakes testing
- Usable mathematics
  - Mathematical literacy, problem solving, modeling

Mathematics & Reality

- Numeracy all around you
- What is Mathematical Literacy?
- Perspectives on mathematics and reality
- Maguire, Terry, & O'Donoghue, John. (2002). A grounded approach to practitioners training in Ireland: Some findings from a National survey of practitioners in Adult Basic Education. In L.O.Johansen & T. Wedege (Eds.), *Proceedings of the 8th International Conference of Adults Learning Mathematics (ALM) (pp. 120-132). Roskilde, Danmark: Hent, UK: Avanti books.*
  - Assessment
- Modeling and problem solving
- Blum, Werner, Galbraith, Peter L., Henn, Hans-Wolfgang, & Niss, Mogens (Eds.). (2007). *Modelling and applications in mathematics education - The 14th ICMI study. New York, USA: Springer Science & Business Media B.V.*
- http://www.icme12.org/data/Abstract_Seoul.doc

Language, images, numeracy

- Suspension of sense making
  - Verschaffel, Lieven, Greer, Brian, Van Dooren, Wim, & Mukhopadhyay, Swapna (Eds.). (2009). *Words and worlds Modelling verbal descriptions of situations. Rotterdam/Boston/Taipei: SensePublishers.*
- Images of numeracy
  - Hoogland, K. (2016). Images of numeracy: Investigating effects of visual representations of problem situations in contextual mathematical problem solving. (PhD-thesis), Technical University Eindhoven, The Netherlands.
- Quick course in changing the representation
  - Exercise 1
  - Exercise 2
  - Exercise 3

Definition Mathematical Literacy

Mathematical literacy is defined (in PISA) as:

the capacity to identify, to understand and to engage in mathematics and make well-founded judgements about the role that mathematics plays, as needed for an individual's current and future private life, occupational life, social life with peers and relatives, and life as a constructive, concerned, and reflective citizen.

OECD (1999). Measuring Student Knowledge and Skills: A new Framework for Assessment, p. 50

Numerate behavior involves:

managing a situation or solving a problem in a real context
everyday life
work
societal
further learning

by responding
identifying or locating
acting upon
interpreting
communicating about

to information about mathematical
quantity & number
dimension & shape
pattern and relationships
data & chance
change

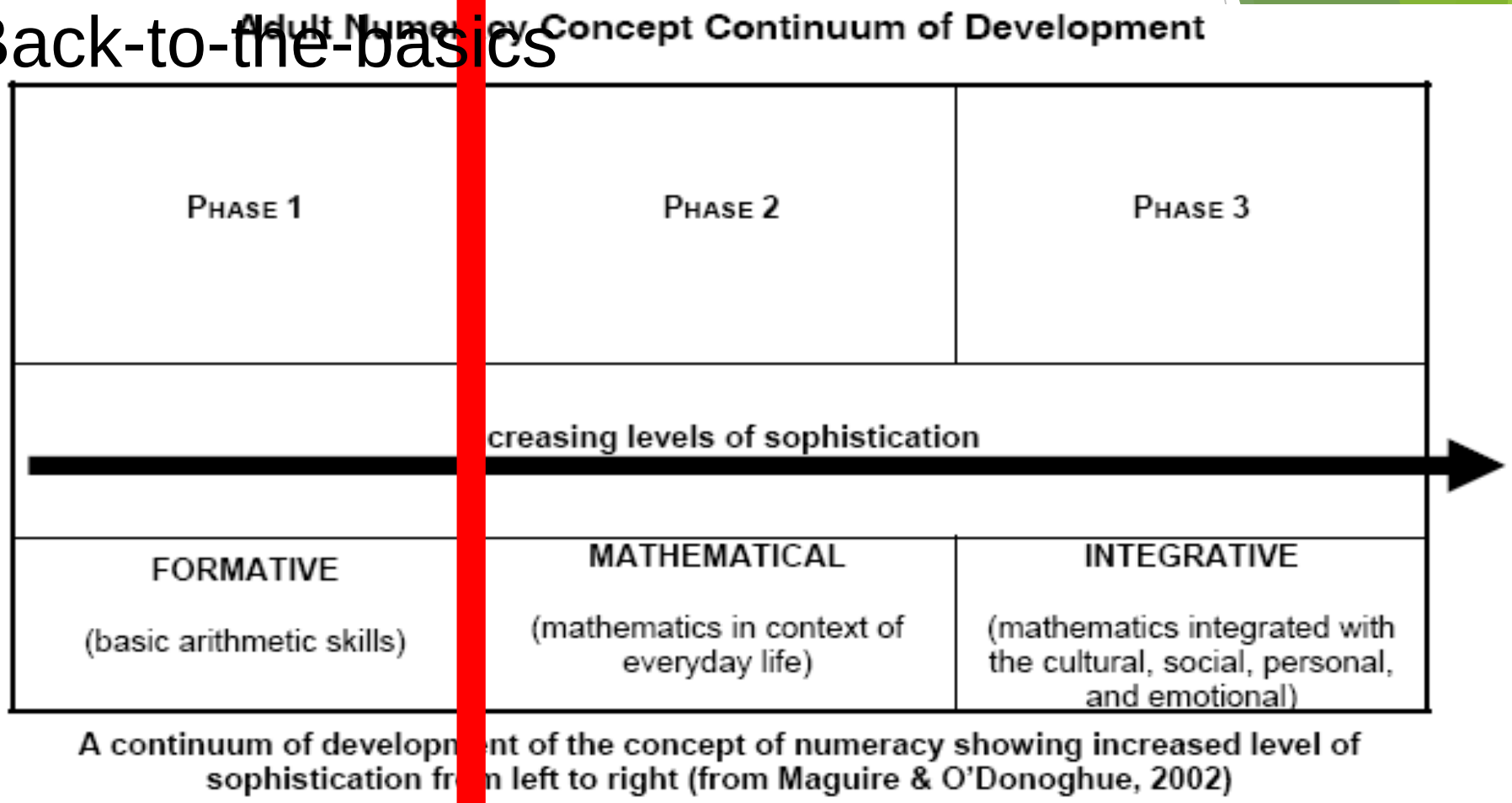
that is represented in a range of ways
objects & pictures
numbers & symbols
formulae
diagrams & maps
graphs
tables
texts

and requires activation of a range of enabling knowledge, behaviors, and processes
mathematical knowledge and understanding
mathematical problem-solving skills
literacy skills
beliefs and attitudes.

ALL
IALS
PIAAC

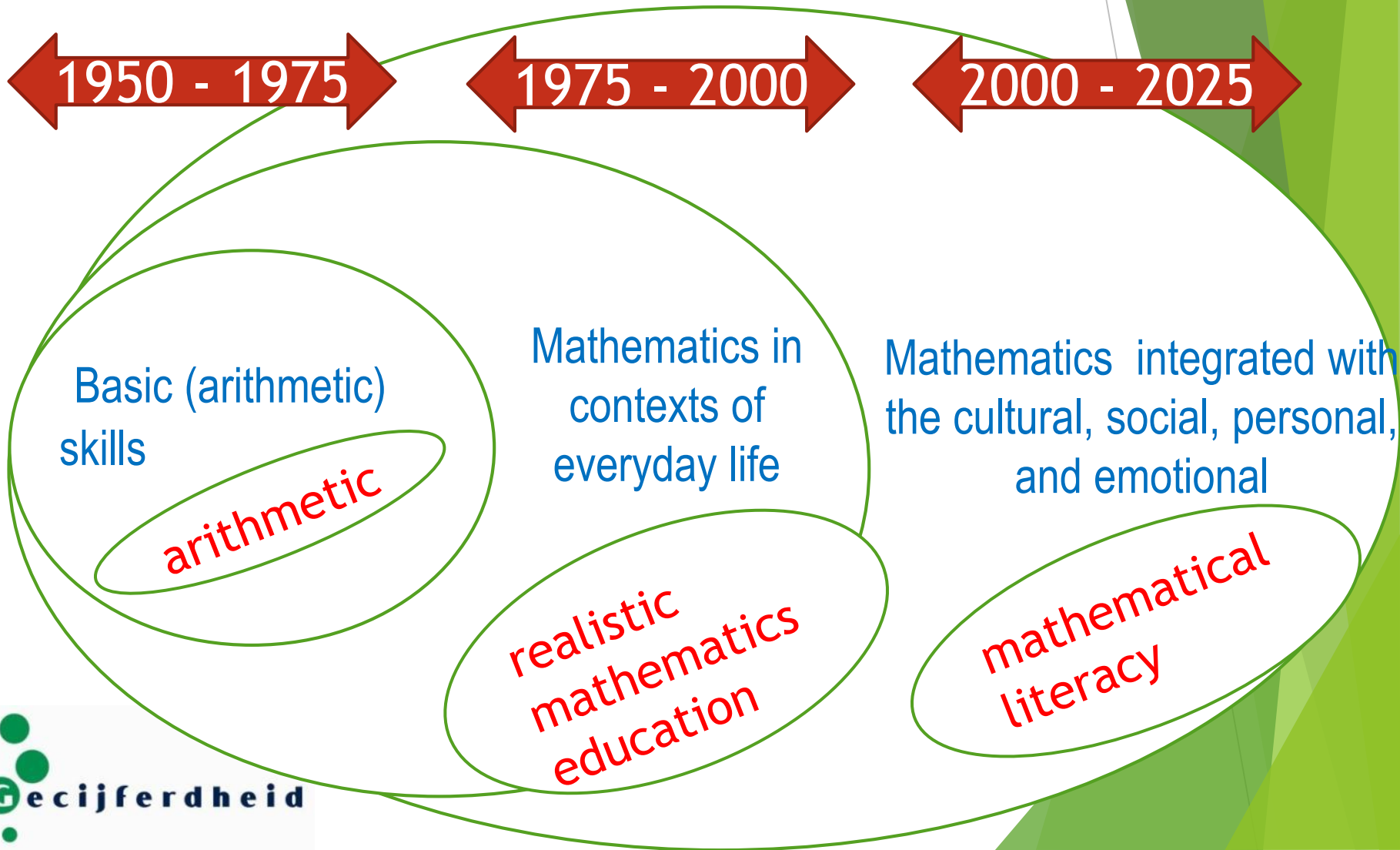
Gal, I, van Groenestijn, M., Manly, M. , Schmitt M.J., and Tout D. (1999) . Numeracy Conceptual Framework for the International Adult Literacy and Lifeskills (ALL) Survey,

Paradigmatic barrier
Epistemological shift
Back-to-the-basics



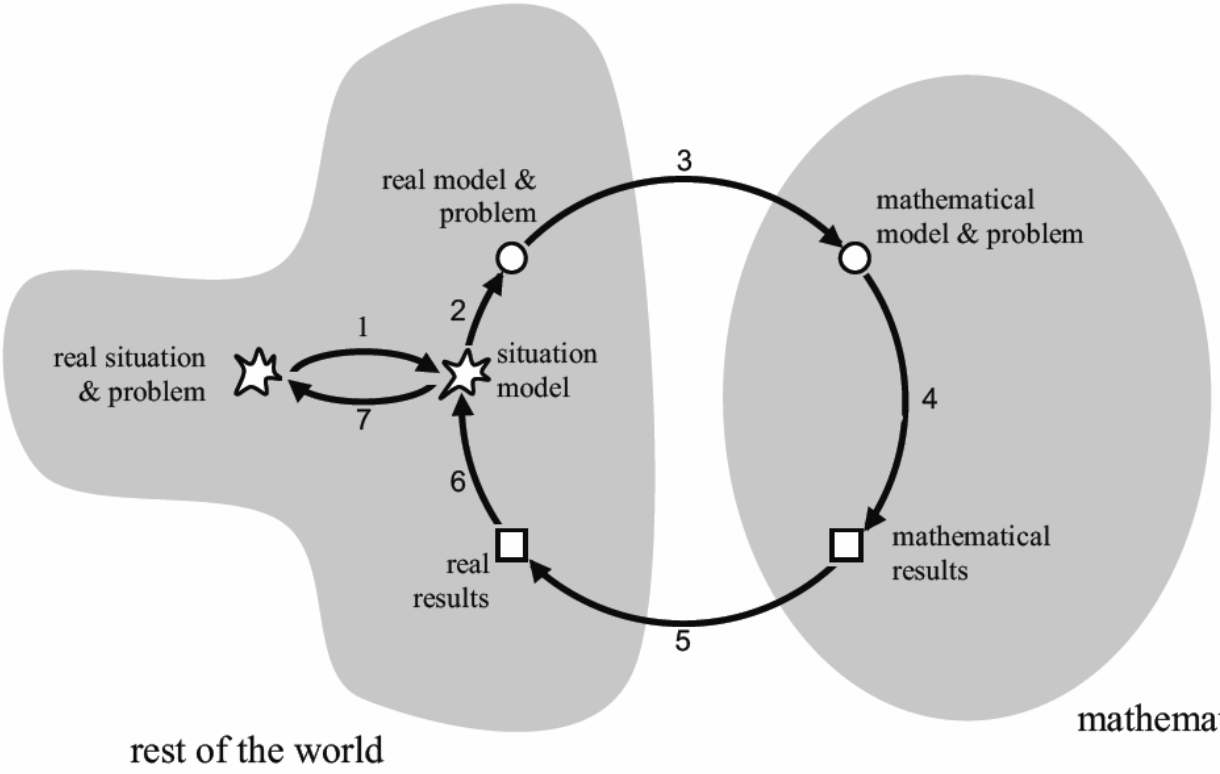
A continuum of develop nt of the concept of numeracy showing increased level of sophistication fr n left to right (from Maguire & O'Donoghue, 2002)

Numeracy & Functional Mathematics
Conceptual development



Gecijferdheid

Different “models” of modelling and problem solving



1 Constructing
2 Simplifying/ Structuring
3 Mathematising
4 Working mathematically
5 Interpreting
6 Validating
7 Exposing

Figure 1 – Modelling cycle

Blum, Werner, Galbraith, Peter L., Henn, Hans-Wolfgang, & Niss, Mogens (Eds.). (2007). *Modelling and applications in mathematics education - The 14th ICMI study. New York, USA: Springer Science & Business Media B.V.*

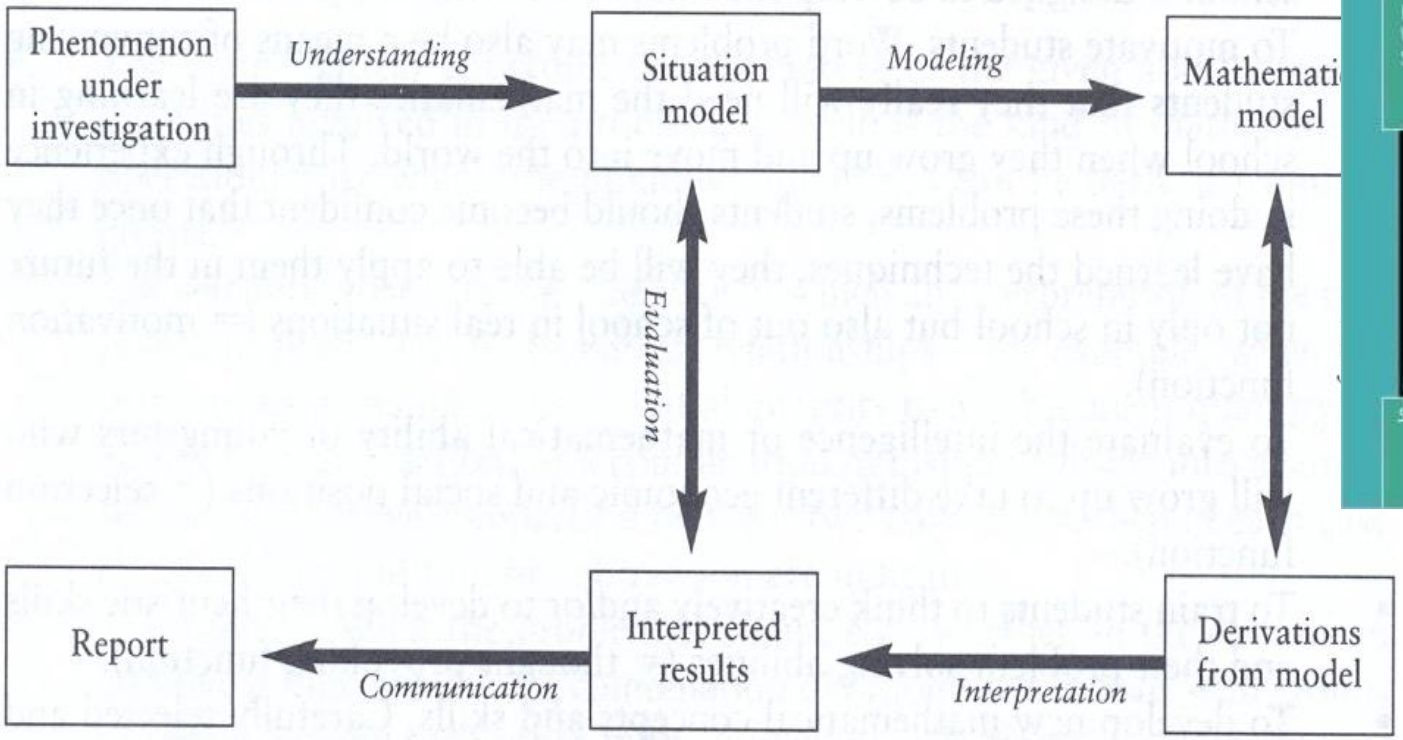
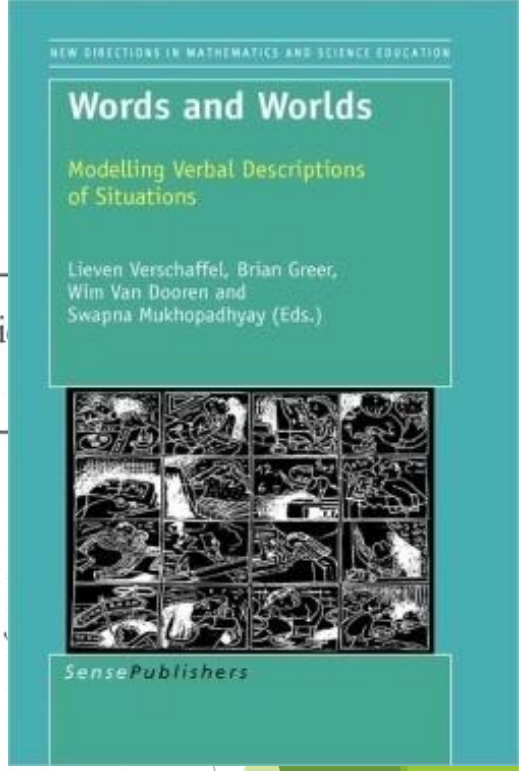


Fig. 0.1 Schematic diagram of the process of modeling.



(Verschaffel , Greer, & De Corte, 2000, p. ix)

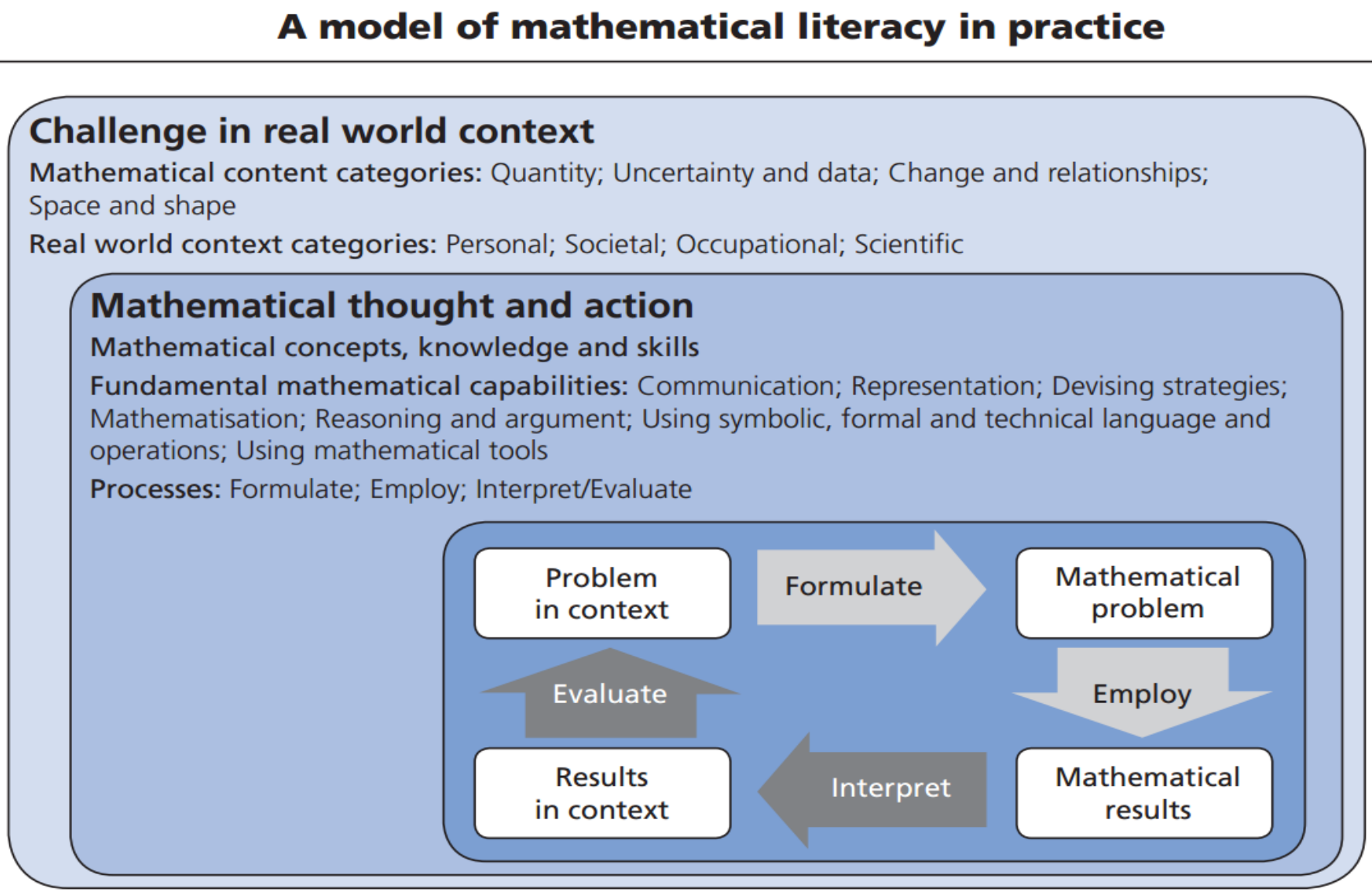
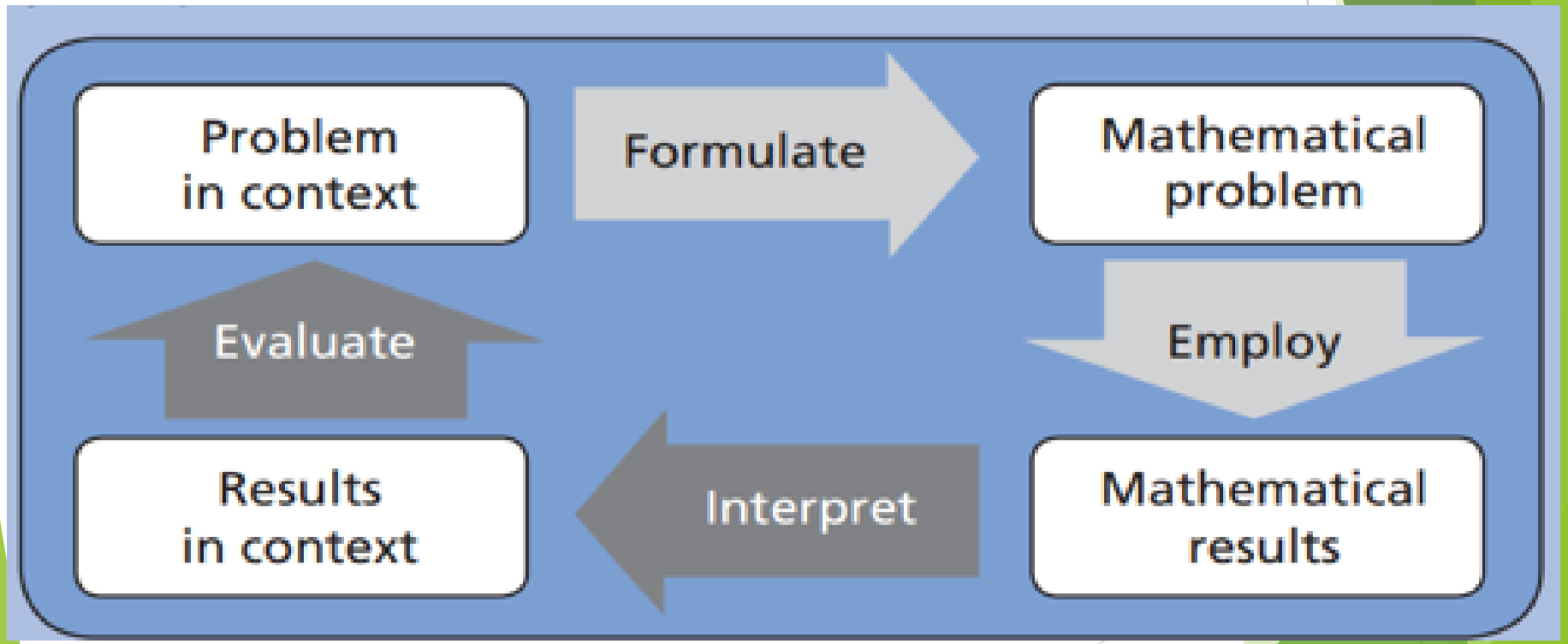


Fig. 1. A model of mathematical literacy in practice. From OECD (2013) (p. 26).

OECD (2013). PISA 2015 draft mathematics framework. Paris, France, OECD Publishing.

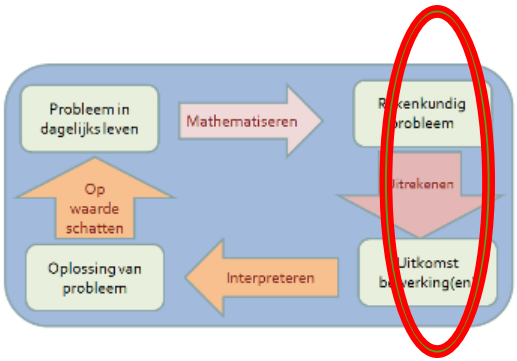
Modeling

Mathematizing



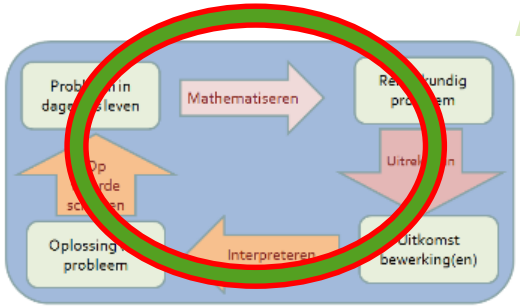
Answer getting

- Only right hand side
- The other parts are just "noise".
- It is all about the operations.



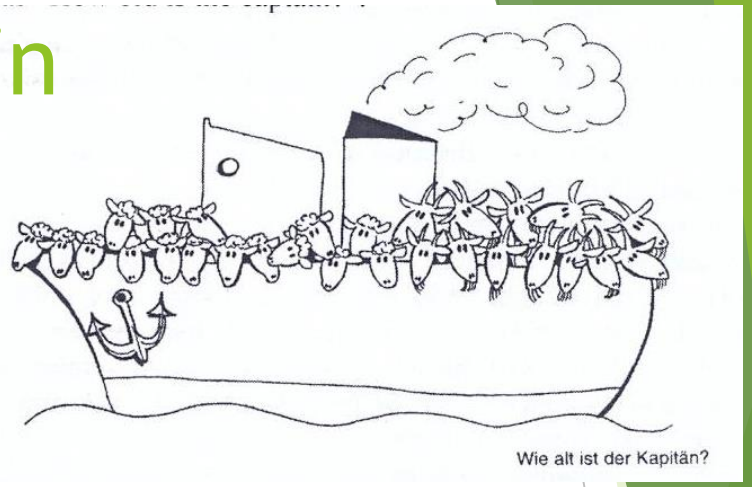
Problem solving

- Always the whole cycle.
- It is an organic system.
- Horizontal steps are the essence of mathematical thinking.



Age of the captain

A captain owns 26 sheep and 10 goats.
How old is the captain?



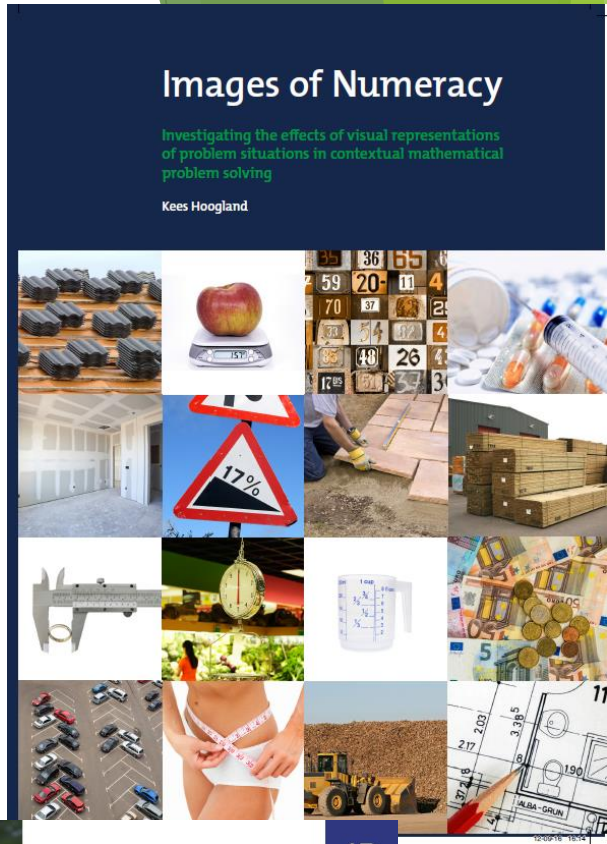
- Suspension of sense making
- Calculational approach

Puisque tu fais de la géométrie et de la trigonométrie, je vais te donner un problème : Un navire est en mer, il est parti de Boston chargé de coton, il jauge 200 tonneaux. Il fait voile vers le Havre, le grand mât est cassé, il y a un mousse sur le gaillard d'avant, les passagers sont au nombre de douze, le vent souffle N.-E.-E., l'horloge marque 3 heures un quart d'après-midi, on est au mois de mai… On demande l'**âge du capitaine** ?
Flaubert, 1841

Images of Numeracy

Investigating the effects of visual representations of problem situations in contextual mathematical problem solving

From a descriptive representation of reality to a depictive representation of reality.



1A

Apples are sold in bags of 2.5 kilograms.
You weigh one apple and find 157 grams.

About how many apples are there in the bag?
☐ **apples**

1B

About how many apples are there in the bag?
☐ **apples**

Design and validation instrument

Table 3.2
Overview of activities undertaken in design of the instrument to counter threats to validity and reliability

Phase of development	Number of research activity	Description of research activity	To counter threats to:
Design	1	Selecting 40 existing items around level 2F of the LaNF.	content validity
	2	Designing 40 alternatives and gathering comments on quality of 40 paired problems by 13 experts.	content validity
	3	Estimation of levels 2F of 40 revised paired problems by eight experts.	construct validity
Validating the diagnostic use	4	Creating a web-based version similar to the nationwide examination in content and layout.	construct validity
	5	Relating the items to the LaNF and spreading the items evenly over the domains of the LaNF.	content validity
	6	Performing a test run with over 7000 participants.	feasibility
	7	Checking for internal consistency of the items with measures of the classical item response theory.	criterion validity
	8	Checking correlation of scores on both versions.	content validity
Validating the measurement of changing the representation of the problem situation	9	Checking 40 revised paired problems on equivalence of paired items by eight experts.	content validity
	10	Programming random representations of the problem situation in 21 items and presenting them in random order in the instrument.	construct validity and reliability
	11	Computer scoring students' solutions.	reliability
	12	After test run: checking for correlation between scores on both versions.	content validity
Composing the final instrument	13	Combine results from all above to construct the final instrument.	

Data

- Main run
  - 179 schools
  - 31,842 students
  - Primary (11-12 yr)
    - 1,150
  - Secondary (12-16/18 yr)
    - 29,500
  - Sec. vocational (16-20 ytr)
    - 1,000

- Collected data
  - Scores on items
  - Answers to items
  - Grade level
  - School track level
  - Age
  - SES
  - Gender
  - Ethnicity
  - Time spent on items in ms
  - Last math grade

Table 4.6

Probit model coefficients and marginal effects of manipulated and non-manipulated variables on school type

Variable	Coefficient	Marginal effect
Version	0.05 (0.01)	.02 (.00)
BO	-0.52*** (0.01)	-.19*** (.00)
VMBO-BB	-0.93*** (0.01)	-.31*** (.00)
VMBO-KB	-0.64*** (0.01)	-.23*** (.00)
VMBO-GT	-0.33*** (0.01)	-.13*** (.00)
HAVO	ref. cat.	ref. cat.
VWO	0.28*** (0.01)	.11*** (.00)
MBO	0.23*** (0.01)	.09*** (.00)
Grade level	0.22*** (0.00)	.09*** (.00)
BO * Version	-0.01 (0.02)	.00 (.01)
VMBO-BB * Version	0.02 (0.02)	.01 (.01)
VMBO-KB * Version	0.03* (0.01)	.01* (.01)
VMBO-GT * Version	0.00 (0.01)	.00 (.00)
HAVO * Version	ref. cat.	ref. cat.
VWO * Version	0.01 (0.01)	.00 (.00)
MBO * Version	0.01 (0.02)	.00 (.01)
Unknown Variables	-0.54*** (0.01)	

Note. Coefficients (and standard errors) and marginal effects (and standard errors) are displayed. Ref. cat. is reference category. Variables are significant *p< .05, **p< .01, ***p< .001, variables' interactions are not significant, with the exception of VMBO-KB * Version.

Results

- B > A statistically significant, with a (very) small effect size.
- B > A on a significant number of problems (11/21)
- Bigger effect in domain of measurement & geometry

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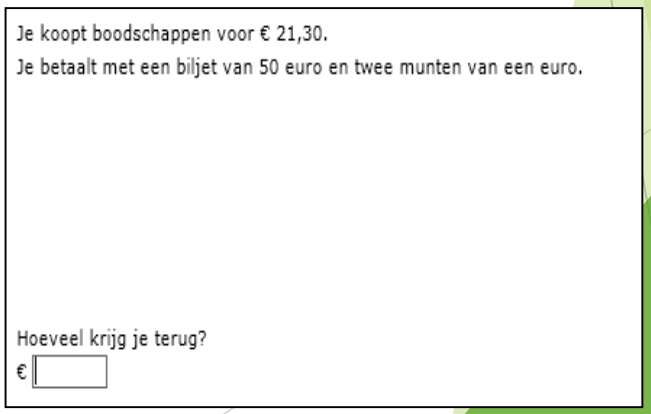

- Further research
  - Interdependency on other variables
  - Actual student behavior
  - Eye-tracking
  - Teaching focus