Beyond Word Problems

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"*Concepts of adult numeracy education can be arranged along a continuum of increasing levels of sophistication. According to a review of AIR (2006) all of the most recent influential approaches to defining adult numeracy fall into the so called integrative phase of this continuum. In this phase numeracy is viewed as a complex multifaceted and sophisticated construct, incorporating the mathematics, communications, cultural, social, emotional and personal aspects of each individual in context.*
A closer look however at lesson or test materials used in many different countries reveals that most materials consist of word problems or of exercises with formal arithmetic skills. One could say that the sophistication of the concepts runs way ahead of the sophistication of the learning and testing materials.
In this era of technological and multimedia possibilities a next step can and should be made to bring real quantitative problems – problems that individuals face or can imagine - into learning materials."

The school curriculum in the Netherlands

In 1985 a brand new curriculum started for upper secondary education divided into two strands: Mathematics A and Mathematics B. Mathematics B was aimed at students with a talent for mathematics and a desire to pursue technical studies or natural sciences. Mathematics A was aimed at students that did not need the rigorous algebra and the abstract formalizations of academic mathematics, but needed some mathematics for everyday life. The latter was a kind of 'mathematical literacy' before the term existed. Mathematics A connected mathematics with the real world, the lesson material was mainly contextual, as were the final examinations. Over the last 25 years I have collected hundreds of clippings from newspapers and magazines, written textbooks on mathematics A and 'invented' dozens of contexts to contextualize the desired mathematics topics. Typically the way I did this was to describe, mainly by usingwords, the context in which the mathematics played a role.

Initially I wrote such descriptions using a typewriter. This was superseded by the desktop PC whereby Wordperfect4.2 and Word were the most important tools to create the connection between mathematics and the real world. I give you a typical example:

A timber dealer buys a number of trees to cut up into planks. The price for a tree depends on the number of cubic feet of wood. There is no easy way to establish

this for a large tree. That is why at a height of 4 feet the circumference of the tree is measured. For each sort of tree a standard price per square centimeter of the crosscut is established.

Calculate the price for an oak with a circumference of 230 cm and a standard price of €2.70 per square cm crosscut.

(source: Moderne wiskunde A1 havo bovenbouw, 2003, translated)

For many years students found this a very refreshing way to do mathematics. In many countries these kinds of contexts have become more and more dominant in lesson materials for mathematics education at all levels. In numeracy education too the word problem is nowadays used regularly as a way to connect mathematics with real life.

Adult numeracy

There are many definitions of numeracy and there are even more opinions on what numeracy education for adults should look like. But there seems to be one point of consensus on numeracy education: it should be functional in everyday life and as a consequence the mathematics should be connected with real life in one way or another. Over the last ten years word problems seem to have become the dominant way to make the connection between mathematics and real life. This, it is argued, has made mathematics much more meaningful for many students of all ages. However we need to ask: is there a problem with the fact that much more language has been introduced into maths lessons? From experience it is clear that having problems with maths, for many students, often coincides with having trouble in coping with written language. So you win some and you lose some. Or not? Are there better ways to make the connection between maths and real life, that are suitable for maths lessons? I would argue that there are.

Using multimedia

It is increasingly possible to bring real life into learning materials by making use of multimedia techniques. If for instance the learning material is web based, you can insert images, video clips, animations, voice-over etc. In recent years some examples of this have been developed in the Netherlands. See for instance the webbased numeracy lesson materials Gecijferd12 en Gecijferd34 on the website www.gecijferd.nl

But even when the learning material is confined to paper much can be done to introduce real life in other ways than by lengthy descriptions of contexts. Careful use of relevant images is the key here. Two examples:

Example: Fridge thermometers

At the top is a question I found in a regular adult numeracy text book. I then show the three steps in which I made some changes to it without losing the maths skills being tested. Below I explain the underlying thinking behind these changes:

I work in a butcher's shop from Monday to Thursday. One of my jobs is to check the temperature of the fridge twice a day. At the right hand side you see a thermometer.
What is the temperature?

A +15° C

B -15° C

C + 15° F

D - 15° F

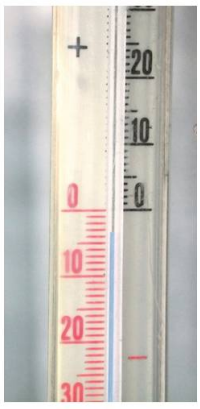
I work in a butcher's shop from Monday to Thursday. One of my jobs is to check the temperature of the fridge twice a day. At the right hand side you see a thermometer.
What is the temperature?

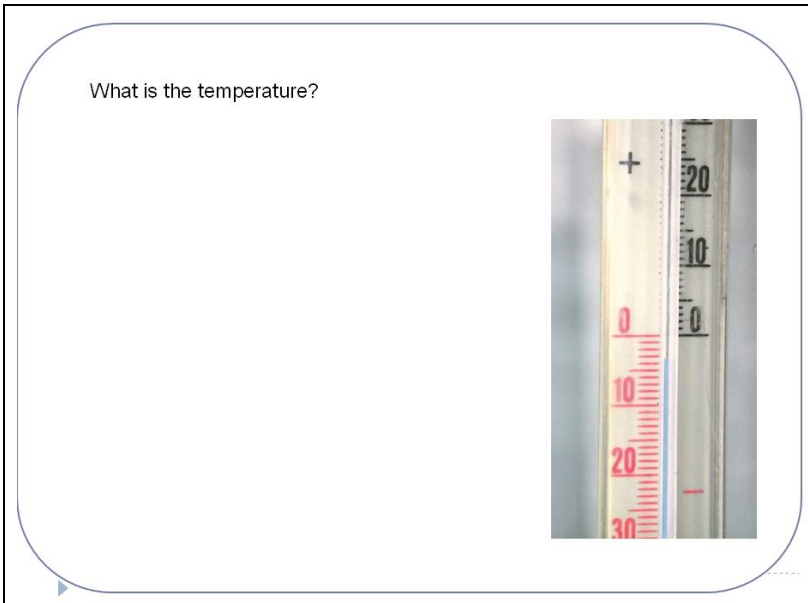
A

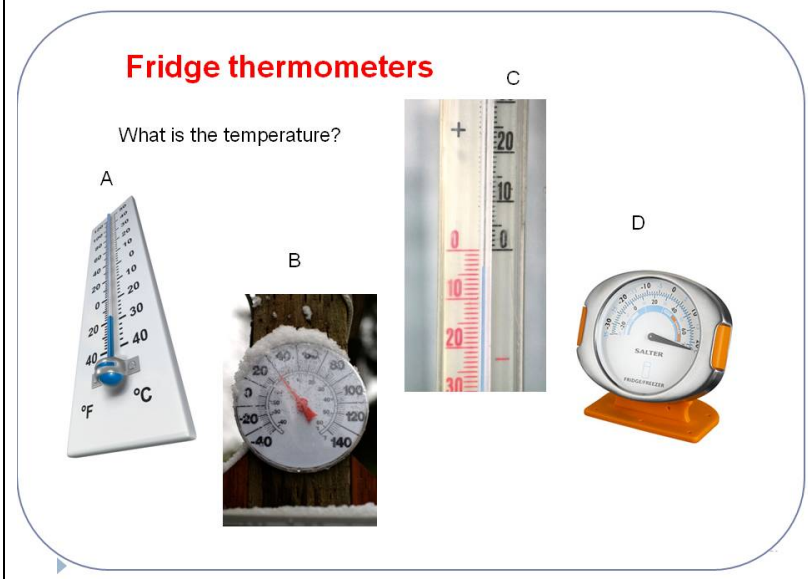
B

C

D







The drawing is replaced with a picture. Drawings only send a message that the mathematics is hidden in a made-up context. Pictures immediately connect the mathematics with the real world. Do not underestimate the effect of such messages on the students' views of the goals of mathematics lessons.

The multiple choice alternatives are nonsensical and debilitating. They are consequently discarded.

The language is brought down to the essence of the problem. Indeed you now lose the background of the actual employee, but describing that background only in words is hardly a benefit. Without it we still have a recognisable question from real life. From the title 'Fridge thermometers' the students can easily imagine a situation where this might be relevant.

If the skill is reading temperatures in fridges or in other situations where temperature measurement is important then the students should be taught to do that. The last image reflects this wider context.

Example: Percentage discounts

In 2008 in the Netherlands a new numeracy framework was published (Expertgroep Doorlopende Leerlijnen Taal en Rekenen, 2008), more commonly known as "the Meijerink report". In this framework three levels of functional numeracy are defined: 1F stands for the functional numeracy at the age of 12, 2F for the functional numeracy at the age of 16 and 3F for the functional numeracy at the age of 19.

The following is a question taken from the first test designed as an example of the levels in this framework ands to illustrate the '2F level' in mathematics. This level of functional numeracy should be within the reach of 85% of the adult population.

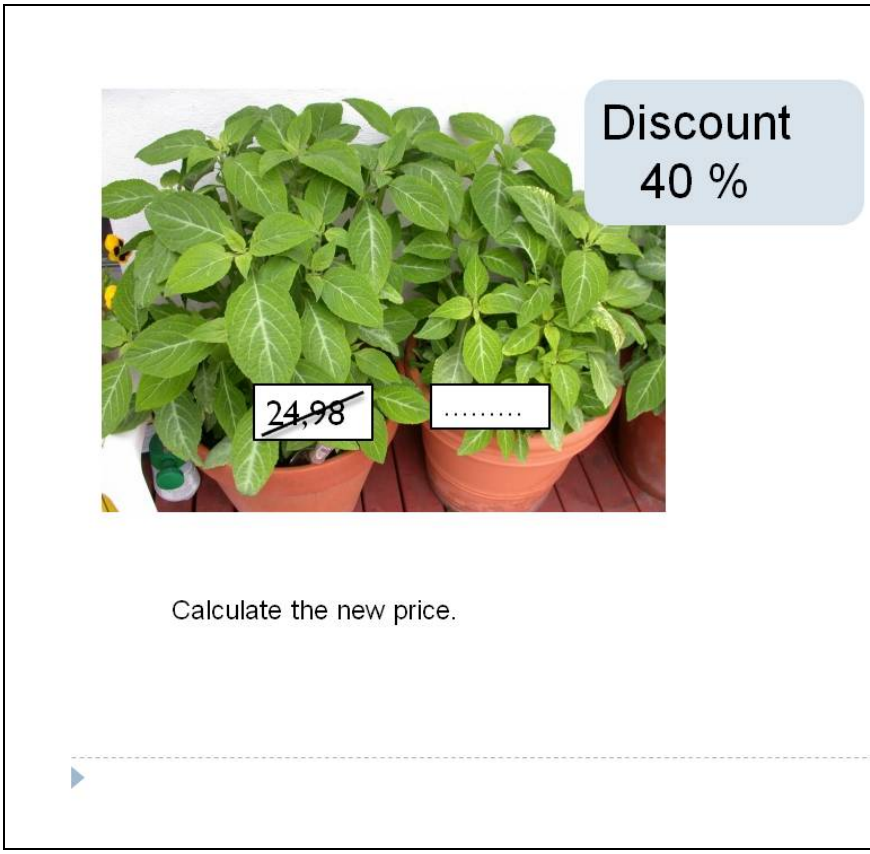
You can guess what the Dutch text means from the drawing but to leave no doubt, it reads: 'What is the approximate percentage discount on this houseplant.'

Opgave 2

Hoeveel procent korting krijg je ongeveer op de kamerplant?

A 40%
B 10%
C 67%
D 60%

Source: Cito (2009). Referentietoetsen 1F/2F



I think this question is fairly representative of current practice in numeracy word problems:

- A drawing instead of a picture, probably because it is cheaper and faster to produce.
- The question repeats unnecessarily in words, a part of the picture.
- The word ‘approximate’ is added to avoid any confusion (it is possible that more confusion is caused by the multiple choices).
- The question is the ‘wrong way’ around – in real life we would rarely want to know this. The question is therefore not ‘imaginable’ as a real life problem.

The alternative is quite simple:

- I know life is all about making choices, but in numeracy education we can do without the silly multiple choice model
- The drawing is replaced with a picture of a real houseplant. Even better if you could have a picture of this houseplant in a garden centre with the real discounts printed or handwritten on a little chalk board.
- The question is imaginable: you can think of a person wanting to perform this task.

- You can expand the question easily by taking pictures of more and different plants.

From these examples we can try to formulate a definition of a numeracy problem that reflects these considerations:

> 'A numeracy problem is an *imaginable* question about the *real world* that can be solved by *mathematical techniques*. A numeracy problem in learning materials uses a *minimum of language* and a *maximum of real life images*. The images are essential for the problem to be solved.'

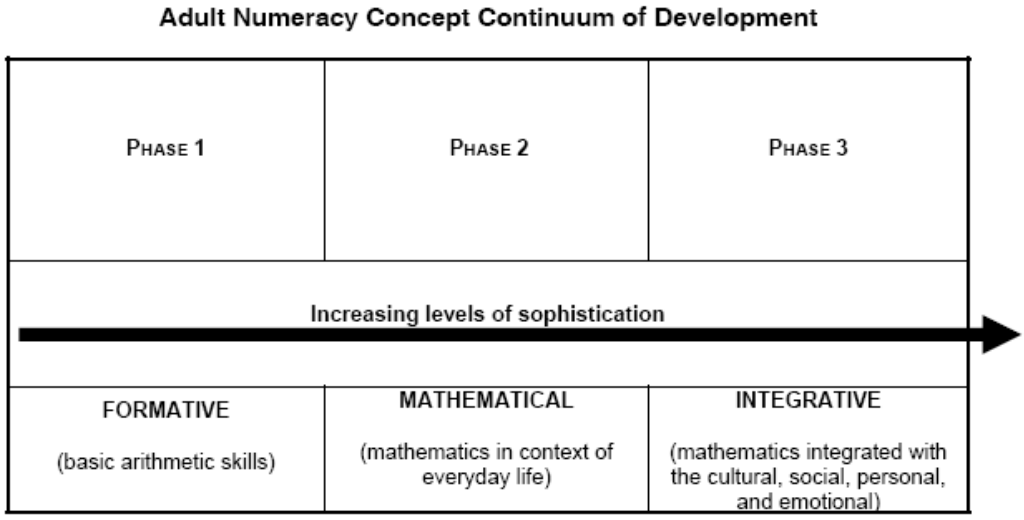
Unavoidable words

But how about contexts that are unavoidably rich in language, for instance tax forms, utility bills, brochures, etc? Of course these should not be avoided. It helps the learner when the real forms and bills are presented as they are, and not as an illustration or a drawing. Of course the official language in forms can be a problem for the students, but it is surely one of the goals of numeracy education to teach the students to cope with that. Which is quite different from the situation where the students have to cope with the language the textbook author has used to encode the real life situation in a description. Dealing with that is hopefully not the goal of numeracy education. Nonetheless, sometimes I encounter systematic problem solving approaches to solve word problems (e.g. 'underline which words are relevant, make a sketch of the situation', etc.). To me this is like harnessing a powerful horse behind a cart with broken wheels.

Trying to make the connection of mathematics to real life more imaginable is also in line with current thinking on approaches to numeracy education.

Some theoretical background

Maguire and O'Donoghue (2002) did some excellent work in creating a framework to give an overview of the various concepts of adult numeracy education being used at present. In their view these concepts can be arranged along a continuum of increasing levels of sophistication.



A continuum of development of the concept of numeracy showing increased level of sophistication from left to right (from Maguire & O'Donoghue, 2002)

The concept of numeracy in the formative phase refers to basic mathematical, or sometimes specifically numerical or quantitative, skills, which adults are deemed to need to function effectively in society. In this view, numeracy is a basic skill normally acquired in childhood; in many numeracy strategies, what adults are deemed to need is simple arithmetic. Lesson material with a strict focus on doing arithmetical operations is widespread and reflects this view.

Numeracy in the mathematical phase is broader, and puts the emphasis on the use of mathematics in daily life. In this phase, numeracy often includes number, money, and percentages; aspects of algebraic, geometric, and statistical thinking; and problem-solving based on the mathematical demands of adult life. This view of numeracy has been quite influential in the Anglophone world, including the United Kingdom's Adult Numeracy Core Curriculum.

All of the most recent approaches to defining adult numeracy fall into Maguire and O'Donoghue's *integrative* phase. In this phase, numeracy is viewed as a complex, multifaceted, and sophisticated construct, incorporating the mathematics, communication, cultural, social, emotional, and personal aspects of each individual in context. These more integrative approaches to numeracy have become influential over the last few years, as illustrated by projects to define numeracy instructional content standards, the Program for International Student Assessment (PISA), and the Adult Literacy and Lifeskills (ALL) Survey. The numeracy definitions in these projects specify the intended cognitive outcomes of adult numeracy education and/or emphasize the need for the individual to adjust to the increasing technological demands of the knowledge economy.

However a closer look at lesson or test materials used in many different countries reveals that most of these still consist of word problems or of exercises with formal arithmetic skills. One could say that the sophistication of the numeracy concepts runs way ahead of the sophistication of the learning and testing materials.

In this era of technological and multimedia possibilities a next step can and should be made to bring real quantitative problems – problems as individuals face them - into learning materials. I hope the examples in this article give practitioners some encouragement to do that.

Challenge

Would it not be wonderful if we could transform most of the existing word problems into numeracy problems in the forthcoming years?

I see it as a challenge to make a start with this. It is reasonably possible to transform any word problem within one day into a numeracy problem. If you send me a word problem (with source) I will send it back transformed in a numeracy problem. You have to give me permission to publish the original and the transformed problem on my website. I have started the process already with some examples which can be found at www.gecijferdheid.nl/transformations

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