Have a basic vocational student perform the PISA assignments and the result will probably be lower than average. Are these students incapable of dealing with numbers? On the contrary, Kees Hoogland believes. You just have to look for the right thing: numerical competences. More gestures than words….

How numerate is a basic vocational student in VMBO (pre-vocational secondary education)?

Introduction

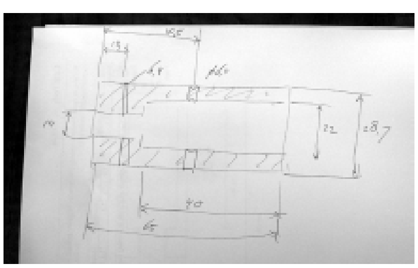
‘You saw off these planks with an excess length of 3 cm, which means that you have 15 mm left on each side.’

‘Look, that 35 x 95 (from the drawing) is here on this side. That means you know that the beam fits exactly in that opening, if you slide it together.’

If those large ginger nuts are ready in 20 minutes, the small ones will be black. I know that, but it doesn’t interest me.’

‘You bend the top of that garden lamp from a sheet. This (the triangle) will always fit, since it (the cap) folds along with it.’

These are examples of student reasoning or student actions in practical situations in which these students use arithmetic or mathematics. These students are in the basic vocational programme of VMBO (pre-vocational secondary education). Their schools are part of the WINST project.[1] The objective of the WINST project is to make education in the technology sector more attractive and accessible by relating the general subjects of mathematics, physics and chemistry to vocational subjects (Abels et al. 2003, Wijers et al. 2004a, 2004b).



My special interest within that project is in the arithmetic or mathematical knowledge and skills that students use in practical situations. In this article, I look at students who are working on practical assignments or tasks. I am fascinated by their abilities in such concrete situations. They show a lot of knowledge and skills. Knowledge and skills that are much less visible in a regular mathematics lesson for these students.

Numeracy and mathematical literacy

I believe that the use of arithmetic or mathematics in concrete practical situations is at the core of what is referred to nowadays as numeracy or mathematical literacy.
The concepts of numeracy and mathematical literacy are rapidly emerging in all sorts of discussions about the development and implementation of arithmetic and mathematics education, both in primary and secondary education. These concepts play a role not only in the Netherlands (Hoogland, 2005) but also around the world (Hoogland & Jablonka, 2003). How do people deal with the quantitative side of the world around us? No one denies that this world is full of all sorts of matters that can be called quantitative or mathematical. But how you organise education to teach people to learn to deal with it in an effective and critical manner is another story entirely. The concepts of numeracy and mathematical literacy are still relatively new, which means that all sorts of definitions are in circulation[2]. I will compare two of these definitions:

Definition 1
Numeracy is the ability to process, interpret and communicate numerical, quantitative, spatial, statistical and even mathematical information in ways that are appropriate for a variety of contexts, and that will enable a typical member of the culture or subculture to participate effectively in activities that they value. (Evans, 2000).

Definition 2
Mathematical literacy is the capacity to identify, to understand, and to engage in mathematics and make well-founded judgements about the role that mathematics plays, as needed for individuals’ current and future private life, occupational life, social life with peers and relatives and as a constructive, concerned and reflective citizen. (OECD, 1999).

The first definition appeals to me the most. It is very broad and covers more than merely the quantitative side. This definition fits well with what I have written about what fascinates me, namely how people interpret and deal with the quantitative and mathematical side of the world in their own activities.
I come across the second definition in many places. Part of the reason is that this definition is at the foundation of the global PISA study. This second definition assumes that mathematics is something beyond us, something that is independent of culture and free of values. The definition argues that we can make a judgement about the role that mathematics plays, as if mathematics were a real existing phenomenon of its own.

Why this article in a PISA special?

PISA measures a form of numeracy or mathematical literacy. The fact that the media continuously talk about a study on student mathematical capacities does not change that. PISA also measures mathematical literacy in a special way, namely by conducting written tests. In the PISA study, tests are presented to students and these students have to show their mathematical knowledge and skills in writing in a written context. The study measures no more and no less than that.
The question is whether in so doing, you also measure whether the student thinks, reasons and deals mathematically in all sorts of practical situations. I do not believe so. I believe you measure student competency in realistic mathematics education. It does not surprise me that we score high in The Netherlands. It is the mathematics education we have been teaching for years. The fact that the Dutch results in this PISA study show a major discrepancy between the results of HAVO/VWO (senior general secondary education/pre-university) students and VMBO students is not surprising either in light of this article.

I believe that there is another mathematics, namely the mathematics in and of the student himself, the mathematics that exists from the knowledge, skills and personal qualities that students use to cope with the quantitative (or mathematical) side of the world around us. I refer to this combination of knowledge, skills and personal qualities with a somewhat fashionable word ‘numerical competences’.

Quest for the numerical competences of students

The basic assumption of our quest is looking at how students perform in practical situations. We made a number of choices.
First: we look at what these students can do; sometimes how they learned it as well. We try to stay far away from emphasising what these students cannot do. This so-called deficiency thinking is rather prevalent in studies on arithmetic and mathematics education, certainly when the study is about people who are to be found in the left half of the Gauss curve as far as cognitive qualities go.
Second: we did not follow the students in mathematics classes, but rather in situations in which they were working on practical assignments or tasks: cooking, making window frames, making trailers, putting together garden lamps, manufacturing spots, etc.
We set to work as follows: we interviewed the students on this activity right after a practical activity, usually with the product close at hand. The questions were not focused on mathematics or arithmetic. Usually we simply asked whether they could explain what they had done. In the analyses, we looked at shown ‘numerical competences’ in the broadest sense of the word. It may involve quantitative issues, spatial or geometric issues or if-then reasoning; all issues that you would consider in the field of numeracy.

Impressions from the quest

The examples I give are not comprehensive. They are merely illustrative and provide an indication of what was noticeable in brief conversations with students on numerical competences.

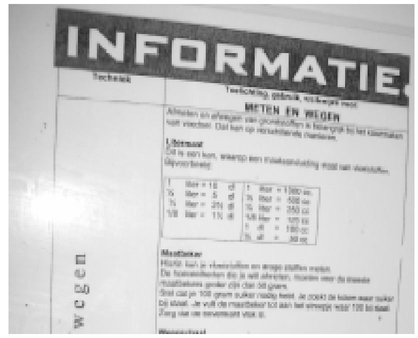
Example 1
Reinder made a window frame from a drawing. He is in the final completion phase. He talks openly and with understanding. He can easily switch back and forth between the drawing and the product. He also has insight into the construction. Reinder explains: ‘These are fixed like so in the wood and then this separates and these are secure and can never be separated again.’
Reinder clearly gesticulates the perpendicular directions and the angle of the force to that. He knows that strength comes from such constructions, where the forces are perpendicular to that. His accompanying gestures are used very precisely to support and visualise.

Example 2
Reinder: ‘First you have to write down everything neatly with your set square. This must be 45 degrees. You do this with your set square. You place it like this, which puts it at exactly 45 degrees.’ 45 degrees is shown on the table with a flat hand.

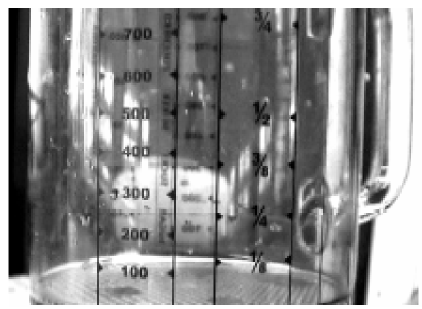
The interviewer inquisitively asks: ‘Exactly 45 degrees right away?’
Reinder answers self-confidently: ‘Yes. It (the set square) has something like that. It goes like this and then there is such a corner in there and you can convert it somewhat, which then makes it exactly 45 degrees’. The workings of the flexible set square are shown with two hands. There is an understanding of angles, how they can be shown and how a device can be adjusted in using it.

Example 3
Cooking is also performed in the Technology and Welfare department.

After the filled speculaas, it is time to make tomato soup. The recipe calls for 2½ decilitres of water.
Lisanne: “Then the teacher said we should look at that piece of paper. Then I looked at that piece of paper and that’s where I saw ¼ of a litre.’



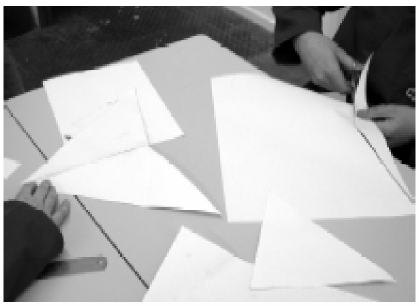
‘And then you have to look at those arrows and add water to the middle of this arrow.’



This student can translate from recipe to list to measuring jug. What is striking here is that is assumed she cannot do it independently. The school solved the problem by hanging up a kind of crib sheet. Apparently, in this lesson, learning that 2½ dl is the same as ¼ of a litre or 250 ml is not an objective, while the measuring jug can be a rich source of discussion with this student.

Example 4
In the Commercial Technology (ComTech) department Zico and a group were assigned to make four garden lamps. They prepared a design after a discussion with the customer. They have seven weeks to design and manufacture. The design phase has just been completed.
Zico explains: “You bend the top of the garden lamp from a sheet. This (the triangle) can be sharp but also wide, it always fits in, like this. Because it (the cap) folds along with it.’

Zico can explain and visualise this well. The interviewer asks about the size of the sheets that are used to make such a lamp. They set the sizes experimentally so they can be made from a 50 x 50 cm sheet. They work a lot with sketches and they draw, cut and paste in advance.



These are just a few examples of numerical competences that you encounter when you look at students who are working on practical assignments or achievements.

Conclusions

It is not the purpose of this article to draw far-reaching conclusions about the organisation of mathematics education to these students, based on a few observations. But a couple of things are striking.

- Students have competences in a large number of areas that we can summarise under numeracy: arithmetic, geometric, if-then reasoning. In each conversation of several minutes about a concrete assignment, a number of these competences can be observed and identified in the manner that I sketched above. Naturally, the question is, how did they acquire these competences? All the students indicated that they learned them from teachers who explained them in a practical situation.
- Students can reason well if they are working in the situation or if they literally have matters in their hands (window frame, spotlight, drawing, measuring jug). When it comes to hypothetical situations or if a concrete situation must be constructed mentally, most of them have a lot more difficulty showing their competences.
- For students, gestures and physical visualisations are important components in their arithmetic and mathematical repertoire. That immediately got me to thinking about the boredom and sterility in doing sums from a book.
- For every question to which these students can answer with a mere yes or no, they do so. With more open questions about how they approached something, these students can reasonably reflect on their actions, thereby showing their competences.

How can you apply this to education?

If I look at this specific target group, students in the basic vocational stream of the VMBO, my conclusions would be:

- Stop doing sums from the book, except when necessary for written tests or exams.
- Work on numerical competences with students in direct and practical situations. How? By taking a good look at what they do in those situations, discussing it with them, having the students explain exactly what they are doing, and having students look at it together and learn from one other.
- Have students collect all sorts of materials and their own productions in numeracy. Collect them in a work folder, file or portfolio and make that the topic of the lesson.

Finally

These impressions are still only based on a number of studies and analyses. In the course of time I hope to learn more about which numerical competences are innate in these students and which are very specifically developed from their own interests.
I have learned that, as an observer, you have to learn to look at what these students do and say to truly acquire a good picture of their capacities and competences. That may be the greatest challenge for teachers, developers and textbook writers.

Kees Hoogland, APS, Utrecht

Notes

[1] For a description of the WINST project see
www.fi.uu.nl/winst
[2] See also the website
www.gecijferdheid.nl

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